§3.2: p. 91: #24.

**Data:**
1. Large tank w/ 500 gal. pure water
2. Brine 2 lb salt/gal @ 5 gal/min
3. Well-stirred
4. Pumped out @ 10 gal/min.

**Required:** Find number lbs of salt in tank @ time \( t \)

\[ A(t) \]

\( \text{when is tank empty?} \)

**Solution**

1. Drug. (Tracking the salt.)

\[ \frac{5 \text{ gal}}{\text{min}} \times \frac{2 \text{ lb salt}}{\text{gal}} = 10 \text{ lb salt/min} \]

\[ 11 \text{ gal out" @ time } t \]

\[ \frac{500 - 5 \text{ gal}}{\text{min}} \]

\[ \text{Out} \]

\[ 10 \text{ gal/min} \times \frac{A(t) \text{ lb salt}}{500 - 5t \text{ gal}} = \frac{2A(t)}{100 - t} \text{ lb salt/min} \]

2. \[ \frac{dA(t)}{dt} = 10 - \frac{2A(t)}{100 - t} \quad \left( \frac{dA}{dt} = R_{\text{IN}} - R_{\text{OUT}} \right) \]

\[ \frac{dA}{dt} + \frac{2}{100 - t} A = 10 \]

Finish at HOME.
§ 4.1 Linear ODE's of HIGHER ORDER.

This section is broken up into 3 subsections.

Now we consider § 4.1.1. (p. 112)

A Main Idea: Thm 4.1: Existence of a unique solution to the IVP.

If
\[ a_n(x) y^{(n)} + a_{n-1}(x) y^{(n-1)} + \ldots + a_1(x) y' + a_0(x) y = q(x) \]

Subject to IC's
\[ y(x_0) = y_0, \ y'(x_0) = y_0', \ldots, y^{(n-1)}(x_0) = y_0^{(n-1)} \]

where \( y_0, y_0', \ldots, y_0^{(n-1)} \) are arbitrary constants and where \( x_0 \) is in the domain of def. of \( y \).

And if \( a_n(x), a_{n-1}(x), \ldots, a_0(x), q(x) \) are all continuous on an interval \( I \) and if \( a_n(x) \neq 0 \) for any \( x \in I \)

Then there exists a unique solution to the IVP.

\[ \exists! y \text{ to the IVP.} \]

B Read Examples in 4.1.1.

C Boundary Value Problems (BVP's) (p. 114)

Example. \( x^2 y'' - xy' + y = 0 \)

Subject to \( y(a) = y_a \) and \( y(b) = y_b \)