§ 5.2: DAMPED MOTION (DM) (p. 192)

A. "In the study of mechanics, damping forces acting on a body are considered to be proportional to a power of the instantaneous velocity."

"... force given by a constant multiple of \( \frac{dx}{dt} \)."

B. Whereas in § 5.1 we had

\[
m \ddot{x} = -kx, \quad (\ast)
\]

if we consider damping, the problem becomes

\[
m \ddot{x} = -kx - \beta \dot{x} \quad (\ast \ast) \quad \text{w/} \quad x(0) = x_0, \quad \dot{x}(0) = v_0
\]

C. Solve (\ast \ast):

\[
m \ddot{x} + \beta \dot{x} + kx = 0
\]

\[
\ddot{x} + \frac{\beta}{m} \dot{x} + \frac{k}{m} x = 0 \quad (\ast \ast \ast)
\]

Let \( \omega^2 = \frac{k}{m} \) \& let \( 2\lambda = \frac{\beta}{m} \)

\[
\therefore (\ast \ast \ast) \text{ becomes}
\]

\[
\ddot{x} + 2\lambda \dot{x} + \omega^2 x = 0 \quad (\ast) \quad \text{STD FORM}
\]

The AuxEq is \( m^2 + 2\lambda m + \omega^2 = 0 \)

Recall in QF \( m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

\[
\Delta = b^2 - 4ac \quad \text{In our case}
\]

\[
\Delta = 4\lambda^2 - 4\omega^2 = 4(\lambda^2 - \omega^2)
\]
The quantity $\lambda^2 - \omega^2 \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$

So we have 3 cases.

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END of CLASS ~~~ AFTER CLASS

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