\[ \mathcal{L}\{\sin(t)\} = \frac{1}{s^2+1} \quad s > 0 \]

We know what the graph of \(\sin(t)\) looks like.

Here is the graph of \(F(s) = \frac{1}{s^2+1}\).

\[ \mathcal{L}\{\sin(kt)\} = \frac{k}{s^2+k^2} \]

\[ \mathcal{L}\{\cos(t)\} = \frac{s}{s^2+k^2} = \frac{s}{s^2+1} \]

\[ \mathcal{L}\{\cos(kt)\} = \frac{s}{s^2+k^2} \]

\[ \mathcal{L}\{e^{at}\} = \frac{1}{s-a} \quad a \in \mathbb{R} \]

Quickies:

\[ \int_{t=0}^{t=b} e^{-st} e^{at} \, dt = \int_{t=0}^{t=b} e^{(a-s)t} \, dt \]

\[ = \left[ \frac{1}{a-s} e^{(a-s)t} \right]_{t=0}^{t=b} = \left[ \frac{1}{s-a} e^{(s-a)t} \right]_{t=0}^{t=b} \]

\[ = \frac{1}{s-a} - \frac{1}{s-a} e^{(s-a)b} \quad -s+a < 0 \quad a < s \]

\[ = \frac{1}{s-a} \left( 1 - e^{(s-a)b} \right) \xrightarrow{b \to \infty} \frac{1}{s-a} \quad \text{as } b \to \infty \]

\[ \therefore \text{This is the confirmation that } \mathcal{L}\{e^{at}\} = \frac{1}{s-a} \text{ if } s > a \]
III. Where Does This Go?

Problem: \( y'' + ay' + by = g \)

\[ \mathcal{L} \rightarrow \text{ALG} \text{ Problem} \]

\[ \mathcal{L}^{-1} \rightarrow \text{ALG Sol.} \]

\[ \text{Sol. to ODE.} \]

Partial Fractions

IV. \( \mathcal{L}\{y'\} \)

First of all \( \mathcal{L}\{y\} = Y \)

\[ \mathcal{L}\{y'\} = sY - y(0) \]

V. Not every function has a Laplace Transform!

A. The functions for which we want Laplace Transforms are the forcing functions (driving functions), \( g(x) \) or \( g(t) \).

B. The good news is that L.T. works for many important piecewise continuous functions.

C. It would be nice if we had some sort of criteria for deciding ahead of time whether a function has a L.T.

And we do. — P. 283.

\[ \text{Exponential Order} - f \text{ is of E.O. if \( f \)} \exists c, M > 0 \text{ and } T > 0 \text{ such that} \]

\[ |f(t)| \leq Me^{ct} \quad \forall t > T \]