11. Find all Real Zeros.

\[2x^3 + 3x^2 - 11x - 6\]

**Solution**

(a) Possible values: \( \pm \frac{1}{1}, \pm \frac{2}{1}, \pm \frac{3}{1}, \pm \frac{6}{1}, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{6}{3} \).

(b) Trial & Error.

Try \( -1 \) \[\begin{array}{c|cc}
2 & 3 & -11 & -6 \\
\hline
& 2 & -1 & \\
\end{array}\]

Try \( 2 \) \[\begin{array}{c|cc}
1 & 2 & 3 & -11 & -6 \\
\hline
& 4 & 14 & 6 \\
& 2 & 7 & 3 & 0 \\
\end{array}\]

\[\therefore \quad P(x) = (x - 2)(2x^2 + 7x + 3)\]

(c) Try to factor \( 2x^2 + 7x + 3 \):

\[= (2x + 1)(x + 3)\]

\[\therefore \quad P(x) = (x - 2)(2x + 1)(x + 3)\]

(d) The Zeros of \( P(x) \) are \( 2, -\frac{1}{2}, -3 \).
#21 Find all Rlt. Zeros.
\[ P(x) = x^5 - x^4 - 2x^3 + 2x^2 + x - 1 \]
**Solution:** Poss. Rlt. Zeros \( \pm \frac{1}{1} \)

**Trial & Error:**
\[
\begin{array}{c|cccc}
| & 1 & 1 & -2 & 2 & 1 & -1 \\
\hline
1 & 1 & 0 & -2 & 0 & 1 \\
\hline
1 & 1 & 0 & 2 & 0 & 1 \;
\hline
1 & 1 & -1 & -1 \\
\hline
-1 & 1 & 1 & 1 & 1 & 1 \\
\hline
1 & 0 & -1 \;
\hline
\end{array}
\]

\[ x = 1 \]

**So:**
\[ P(x) = (x-1)^2(x+1)(x^2-1) \]

**Factor:**
\[ x^2 - 1 = (x+1)(x-1) \]
so
\[ P(x) = (x-1)^3(x+1)^2 \]

**The Zeros of** \( P(x) \) **are** 1 **multiplicity 3** and -1 **multiplicity 2.**

#31 L.D.
\[ \frac{2x^5 - 7x^4 - 13}{4x^2 - 6x + 8} \]
**Solution:**
\[
\frac{\frac{1}{2}x^3 - x^2 - \frac{5}{2}x - \frac{3}{4}}{4x^2 - 6x + 8}
\]
\[
\frac{2x^5 - 7x^4 + 0x^3 + 0x^2 + 0x - 13}{2x^5 + 3x^4 + 4x^3 + 4x^4 + 6x^3 + 8x^2 + 10x^3 + 15x^2 + 20x - 13}
\]
\[
\begin{array}{c}
\frac{7x^2 + 20x}{-7x^2 + \frac{1}{2}x + 14}
\end{array}
\]
\[
\frac{19}{2}x + 1
\]
#3 cont.

2. Thus

\[
\frac{2x^5 - 7x^4 - 13}{4x^2 - 6x + 8} = \frac{1}{2}x^3 - x^2 - \frac{5}{2}x - \frac{7}{4} + \frac{10x + 1}{4x^2 - 6x + 8}
\]

#4
Use SD & Rem. Thm.: P(6) if

\[P(x) = 2x^3 - 21x^2 + 9x - 200, \quad c = 11.
\]

\[\text{Sol. } [11 \mid 2 -21 9 -200 \overline{22 11 220}}
\]

\[\begin{array}{c}
2 \\
1 \\
20
\end{array}
\]

\[\begin{array}{c}
20
\end{array}
\]

\[\therefore P(11) = 20\]

#5
Evaluate \(\sqrt{-2} \cdot \sqrt{-50}\)

\[\text{Sol. } \sqrt{-2} \cdot \sqrt{-50} = i\sqrt{2} \cdot i\sqrt{50} = i^2\sqrt{100} = -10
\]

(or \(-10 + 0i\))

#6
Evaluate \(\frac{1}{2-i} - \frac{1}{2+i}\)

\[\text{Sol. } \frac{1}{2-i} - \frac{1}{2+i} = \frac{(2+i)-(2-i)}{4+1}
\]

\[\begin{array}{c}
\frac{2i}{5} \\
0 + \frac{2i}{5}
\end{array}
\]

#7
Solve \(2x + \frac{3}{x} = 2\)

\[\text{Sol. } [1 \mid 2x^2 + 3 = 2x \quad a = 2
\]

\[\text{2} \quad 2x^2 - 2x + 3 = 0 \quad \text{QF } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad b = -2
\]

\[\text{c} = +3
\]

\[x = \frac{2 \pm \sqrt{4 - 4(2)(3)}}{4} = \frac{2 \pm 2\sqrt{-5}}{4} = \frac{1 \pm i\sqrt{5}}{2}
\]

\[\left\{ \frac{1}{2} + \frac{\sqrt{5}i}{2}, \frac{1}{2} - \frac{\sqrt{5}i}{2} \right\}
\]
#8 Simp. $i^{99}$  
$= i^{3}$  
$= -i$

#9 Divide $\frac{1-3i}{4+i}$  
$\text{Soln.}$  
$\frac{1-3i}{4+i} \cdot \frac{4-i}{4-i} = \frac{4-13i-3}{17} = \frac{1-13i}{17} = \boxed{\frac{1}{17} - \frac{13}{17}i}$

#10 Factor Completely  
$x^3-x^2+25x-25 = P(x)$  
$\text{Soln. (Short Way)}$  
1) $P(x) = x^3-x^2+25x-25 = x^2(x-1) + 25(x-1)$  
$= (x-1)(x^2+25) = (x-1)(x+5i)(x-5i)$  
2) Zeros are 1, -5i, 5i each w/ mult. 1.

#11 End behavior of $P(x) = -x^5 + 2x^4 - 3x^3 - x^2 + 12$  
$\text{Soln.}$ No work Required.  
$y \to -\infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$

#12 Eval. $(-7+3i)-(3-9i) = -7+3i-3+9i = -10+12i$

#13 SD. $\frac{x^3-x^2-2x+6}{x-2}$  
$\text{Soln.}$  
2)  
$\frac{1}{1} -1 -2 6$  
$\frac{2}{2} \frac{2}{0}$  
$\boxed{1 1 0 6}$
#14 Find all zeros using s.d. + QF.

\[ P(x) = x^3 + 6x^2 + 13x + 10 \]

\[ \text{Sol.:} \quad 1, -2, 5, 10 \]

2. Since all the signs in \( P(x) \) are pos, how could there be a pos. zero?

3. Mentally tried -1. It didn't work.

4. Try \(-2\)

\[ \begin{array}{c|cccc}
   & 1 & 6 & 13 & 10 \\
\hline
-2 & 2 & -8 & 20 \\
\hline
1 & 2 & 5 & 0
\end{array} \]

It worked!

5. \[ P(x) = (x+2)(x^2+4x+5) \]

and I see that the quadratic factor does not factor.

6. You might use the Quad. Formula here, but I'm going to complete the square.

\[ x^2 + 4x + 5 = 0 \Rightarrow x^2 + 4x = -5 \]
\[ \Rightarrow x^2 + 4x + 4 = -5 + 4 \Rightarrow (x+2)^2 = -1 \]
\[ \Rightarrow x+2 = \pm \sqrt{-1} = \pm i \]

7. The zeros are \(-2, -2+i, -2-i\).

#15 deg 3 poly w/ zeros \(2, 1+3i\).

Sol.: \[ 2x \text{They give me 2 zeros. I need 3 to play this game. What is the 3rd zero? ANS.} \]

\[ 1-3i \]

\[ P(x) = (x-2)(x-(1+3i))(x-(1-3i)) \]
\[ = (x-2)(x-1-3i)(x-1+3i) \]
\[ = (x-2)((x-1)-3i)((x-1)+3i) = (x-2)(x-1)^2 - (3i)^2 \]
\[ = (x-2)[x^2 - 2x + 1 + 9] = (x-2)(x^2 - 2x + 10) \]
\[ = x^3 - 2x^2 + 10x - 2x^2 + 4x - 20. \]

3. \[ |P(x)| = x^3 - 4x^2 + 14x - 20 \]

END.