Blackboard Bonus (answered by Debra): Find the Slant Asymptote 
\[ r(x) = \frac{5x^2 - 3x + 2}{x - 7} \]

Solution: \[ 7 \begin{array}{c|c} 5 & -3 & 2 \\ \hline 3 & 2 & 28 \\ \hline & 2 & 28 \\ \end{array} \]
\[ \therefore \text{SA } y = 5x + 32 \]

\[ \text{§ 4.2: Logarithmic Functions. (p. 342)} \]

A. What is a logarithm.

Base "a" \(a > 0\) and \(a \neq 1\)

Define \[ f(x) = \log_a(x) \]

and \(y = \log_a(x)\) \(\iff\) \(a^y = x\)

\(\text{logarithmic form}\)

\(\text{exponential form}\)


1. If \(m\) is a pos. whole number,
\[ a^m = a \times a \times a \times \ldots \times a \]
\(\text{def } n\text{ of them}\)

2. \(a^m \cdot a^n = a^{(m+n)}\)
\(\text{example } a^3 \cdot a^5 = a^8\)

3. \(a^{-m} = \frac{1}{a^m}\)

4. \(a^m \cdot b^m = (ab)^m\)

5. \(\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\)

Know your Rules of Exponents!
**Log Skill: log form ↔ exp form.**

1. \( \log_2 8 = 3 \) \( \Rightarrow \) \( 2^3 = 8 \)

2. \( \log_4 2 = \frac{1}{2} \) \( \Rightarrow \) \( 4^{\frac{1}{2}} = 2 \)

   (recall \( 4^{\frac{1}{2}} = \sqrt{4} \))

3. \( 3^5 = 243 \) \( \log_3 243 = 5 \)

**Log Skill: Solving.**

1. \( \log_2 32 = ? \)

   \[ \text{Solv.} \quad \log_2 32 = x \leftarrow \]

   \[ \Rightarrow 2^x = 32 \quad \text{so} \quad x = 5 \]

   (\( b^c \cdot 2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32 \checkmark \))

   \[ \therefore \log_2 32 = 5 \]

2. \( \log_x 9 = 81 \) Solve for \( x \)

   \[ \text{Solv.} \quad \log_x 9 = 81 \xrightarrow{\text{exp.}} x^{81} = 9 \quad ?? \]

3. \( \log_x 9 = \frac{1}{2} \) \( \Rightarrow \) \( x^{\frac{1}{2}} = 9 \)

   \( \therefore x = 81 \)

   Check: \( \log_{81} 9 = \frac{1}{2} \)

   \( 81^{\frac{1}{2}} = 9 \checkmark \)
The Most Common bases of Logarithms are 10 and \( e \approx 2.718 \).
(Third most common is 2).

1. \( y = \log_{10} x \) (means \( 10^y = x \))
   we write \( y = \log x \)

2. \( y = \log_e x \) (means \( e^y = x \))
   we write \( y = \ln x \)

Universal Properties of Logarithms (to any legal base) (p. 343)

1. \( \log_a 1 = 0 \) \( (a^0 = 1) \)
2. \( \log_a a = 1 \) \( (a^1 = a) \)
3. \( \log_a a^x = x \) \( (a^x = a^x) \)
4. \( a^{\log_a x} = x \) \( (\text{in log form } \rightarrow \log_a x = \log_a x) \)

§ 4.3: Laws of Logarithms, (p. 352)

A3 Laws.

1. **Product Rule** \( \log_a (AB) = \log_a A + \log_a B \)
2. **Quotient Rule** \( \log_a \left(\frac{A}{B}\right) = \log_a A - \log_a B \)
3. **Power Rule** \( \log_a (A^c) = c \log_a A \)
B. "COB" Rule (Change Of Base)

1. \( \log_b x = \frac{\log_a x}{\log_a b} \)

2. Example: \( \log_2 17 = \frac{\log 17}{\log 2} \approx 4.087462841 \uparrow \)
   
   (Note \( \log_2 16 = 4 \), \( 6^4 = 16 \), and 17 is just a little more than 16 so \( \log_2 17 \) should be "just a little more" than \( \log_2 16 \) — And we see that it is!)

IV. Note: also in class we graphed \( y = \log x \) on the TI-84 and discussed several of the properties of the log function:

- \( \log \) (neg number) is not defined.
- \( \log 1 = 0 \)
- \( \log 100 = 2 \)
- \( \log 1000 = 3 \)
- \( \log (1,000,000) = 6 \)

- \( \log \) is an increasing function: \( 0 < a < b \Rightarrow \log a < \log b \).