
A #24 \[ \frac{10}{1 + e^{-x}} = 2 \]

Sol. Correct to 4 decimals.

1 Clear frac. \[ 10 = 2(1 + e^{-x}) \]

Simp. \[ 5 = 1 + e^{-x} \quad \therefore \quad 4 = e^{-x} \]

\[ \therefore \quad 4 = \frac{1}{e^x} \Rightarrow 4e^x = 1 \Rightarrow e^x = \frac{1}{4} \]

2 Solve for \( x \) by "taking" the logarithm of b.s.

\[ \ln(e^x) = \ln\left(\frac{1}{4}\right) \]

\[ x = \ln\left(\frac{1}{4}\right) = \ln 1 - \ln 4 = -\ln 4 \]

3 \[ \left\{ \ln \frac{1}{4} \right\} \quad \text{or} \quad \left\{ -\ln 4 \right\} \quad \text{EXACT} \]

4 Calc Approx. \( x \approx -1.386294361 \)

5 \[ \left\{ -1.3862 \right\} \]

§ 4.5: Modeling, p. 369.

A We'll do

1 Exponential Growth / Decay (p. 370 / p. 373)
2 Richter Scale (log model) (p. 376)
3 pH scale (log) (p. 376).

B Exponential Growth.

1 Equation plus some constraints on the input variable.
2. Model.  \( n(t) = n_0 e^{rt} \)

- \( n \) is population (\( n = \) "number") at time \( t \).
- \( t \) is time.
- \( r \) is relative rate of growth (growth rate).
- Usu. given as a percent, \( 5\% = 0.05 \).
- \( r = 0.05 \).

- \( n_0 \) is the beginning pop.  \( n_0 = n(0) \)

3. You read the problem.  You understand what the \( t \), \( r \), and \( n_0 \) are for that problem.  You plug & chug.

4. Example: §4.5: p. 379: #1.  Answer a bunch of questions about this model. (bact.)

- \( n(t) = 500 e^{0.45t} \)  \( (t \text{ in hours}) \)
- \( n \) is number of bact.

Q.1. Initial number of bact.

\[ 500 \]  There are initially 500 bact. present.

Q.2. Rel. growth rate (as percentage)

The relative growth rate is 45%.

Q.3. How many bact. after 3 hrs.

\[ n(3) = 500 e^{0.45(3)} \approx 1928.712765 \]

After 3 hrs there are approximately 1929 bact. in population.
In how many hours will the population reach 10,000?

\[ n(t) = 500e^{0.45t} \]

We must find \( t \).

Solve Gen. Eq. for \( t \):

\[ n = 500e^{0.45t} \]

\[ \frac{n}{500} = e^{0.45t} \]

\( \text{Take ln} \)

\[ \ln \left( \frac{n}{500} \right) = \ln(e^{0.45t}) = 0.45t \cdot \ln(e) \]

\[ = 0.45t \cdot 1 \implies 0.45t = \ln \left( \frac{n}{500} \right) \]

\[ t = \frac{1}{0.45} \cdot \ln \left( \frac{n}{500} \right) \]

So if \( n = 10000 \),

\[ t = \frac{1}{0.45} \ln \left( \frac{10000}{500} \right) \]

\[ t = \frac{1}{0.45} \ln(20) \approx 6.657 \text{ hrs} \text{ or } 6 \text{ hrs } 36 \text{ min} \]

The population reaches 10,000 in just over 6.6 hrs or just over 6 hrs 36 min.