Suppose an auditorium has 25 seats in the first row, 28 seats in the second row, 31 seats in the third row, and so on. If there are 30 rows in the auditorium, how many seats are there?

\[
\text{So}\begin{align*}
\text{[A.S.]} \quad & \alpha \text{ and } d \\
\alpha &= 25 \quad & n &= 30 \quad d = 3
\end{align*}
\]

\[
S_n = \frac{n}{2} [2\alpha + (n-1)d]
\]

\[
S_{30} = 15 [50 + 29 \cdot 3] = 15 [137] = 2055
\]

There are 2055 seats in the auditorium.

Note: This was an 11.2 question & it could very easily be on your test.

\[\text{II \ § 11.3: Geometric Sequences. (p.838). (GS)}\]

A In a GS we progress from one term to the next by multiplication NOT addition. So we have a "common ratio" rather than a "common difference" to deal with. We call the common ratio "r" not "d", so our important letters are \(\alpha \text{ and } r\) not \(\alpha \text{ and } d\).
B. I'll tell you right now that 

3, 6, 12, 24, 48

is a GS. 

Find "ran" 

\[ r = 2 \quad n = 5 \quad a = 3 \] 

"How many?"

To find \( r \) take any term and put it over the previous term — like this \( \frac{48}{24} = 2 \)

\[ \frac{24}{12} = 2 \quad \frac{6}{3} = 2 \] 

Common Ratio \( r = 2 \)

C. Formula for \( n \)th term of a GS.

\[ a, ar, ar^2, ar^3, ar^4, \ldots \]

\[ a_1, a_2, a_3, a_4, a_5, \ldots \]

\[ a, ar, ar^2, ar^3, ar^4, \ldots \]

\[ a_n = ar^{n-1} \]

D. If \( a = 3 \) and \( r = \frac{1}{2} \) Find \( a_8 \)

Solution: \[ \text{ran} \quad r = \frac{1}{2} \quad a = 3 \quad n = 8 \]

\[ a_n = ar^{n-1} \quad \text{so} \quad a_8 = 3 \cdot \left( \frac{1}{2} \right)^7 = 3 \cdot \frac{1}{128} \]

\( a_8 = \frac{3}{128} \)
The $n$-th partial sum of an GS, $S_n$.

$S_1 = a$
$S_2 = a + ar$
$S_3 = a + ar + ar^2$
$S_4 = a + ar + ar^2 + ar^3$

$S_n = a + ar + \ldots + ar^{n-1}$

\[ S_n = a + ar + ar^2 + \ldots + ar^{n-1} \]

\[ S_n - a = ar + ar^2 + \ldots + ar^{n-1} \]

\[ S_n - a = r(a + ar + ar^2 + \ldots + ar^{n-2}) \]

\[ \frac{S_n - a}{r} = a + ar + ar^2 + \ldots + ar^{n-2} \]

\[ \frac{S_n - a}{r} + ar^{n-1} = a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1} \]

Subs. on right

Now solve for $S_n$. 

\[ \frac{S_n - a + ar^n}{r} = S_n \]
\[ S_n = a + ar^n = rS_n \]
\[ S_n - rS_n = a - ar^n \]
\[ S_n (1 - r) = a (1 - r^n) \]
\[ S_n = \frac{a (1 - r^n)}{1 - r} \]

Formula for the \( n \)-th Partial Sum of a GS.

\[ \square \] In a GS, the first term is 8, the second term is 4. Find the 5\(^{th}\) term (\#33, p. 844).

\[ \text{Soln.} \quad \square \] GS. \( r = \frac{1}{2} \), \( a = 8 \), \( n = 5 \)

\[ 2 \quad a_n = ar^{n-1} \]
\[ a_5 = 8 \left( \frac{1}{2} \right)^4 = 8 \cdot \frac{1}{16} = \frac{1}{2} \]