
A

(Cont. from yesterday).

An "element" of volume

\[ V_k = A(x_k) \Delta x_k \]

\[ V \approx \sum_{k=1}^{n} A(x_k) \Delta x_k \]

\[ V = \lim_{n \to \infty} \sum_{k=1}^{n} A(x_k) \Delta x_k = \int_{x=a}^{x=b} A(x) \, dx \]

Main Idea of § 6.1.

B

Steps.

#1 Sketch

#2 Find your \( \Delta \): \( \Delta x \) or \( \Delta y \)

#3 Find your \([a,b]\) on x-axis & \([c,d]\) on y-axis

#4 Find \( A(x) \)

#5 "Build": \( V = \int_{x=a}^{x=b} A(x) \, dx \) or

\[ V = \int_{y=c}^{y=d} A(y) \, dy \], as the case may be.

C

Cavalieri's Principle: (p. 427).

Solids with equal altitudes (heights) and identical cross-sections at each height level have the same volume.
Solids of Revolution (p. 428)

1. Rot. abt. x-axis \( \Delta x \)
   \[
   V = \int_{x=a}^{x=b} A(x) \, dx \quad [\text{my } A \text{ is a circle}]
   \]
   \[
   V = \int_{x=a}^{x=b} \pi [R(x)]^2 \, dx
   \]

2. Rot. abt. Horiz. Line: \( y = k \)
   \[
   V = \int_{x=a}^{x=b} \pi [R(x)]^2 \, dx
   \]
   be careful calculating \( R(x) \)

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After Class:

1. Here's a fun-one: Find the volume of the solid of rotation:
   \( y = 4x - x^2 \), with \( 1 \leq x \leq 3 \) is rotated about the horizontal line \( y = 3 \).

   Soln.
   1 Sketch
   2 The \( \Delta \) is \( \Delta x \)
   3 The interval: \( y = 4x - x^2 \) and \( y = 3 \) (Solve simultaneously)
      \[
      4x - x^2 = 3
      \]
      \[
      \therefore -x^2 + 4x - 3 = 0
      \]
      \[
      \therefore x^2 - 4x + 3 = 0
      \]
      \[
      \therefore (x-1)(x-3) = 0
      \]
      \[
      \therefore x = 1 \text{ or } x = 3
      \]
   4 \( A(x) \) the area of the (circular) cross-section. Find the (variable) radius, \( R(x) \).

   Just Do It! Finish if you can...