II. Example: \[ \text{Find the vol. of the solid gen. by revolving the region abt. the y-axis.} \]
Region enclosed by triangle w/ vertices \((1,0), (2,1), (1,1)\)

Solution:

2. ID. \( \Delta: \{(x,y)\} \)
3. Limits of Integration (interval) on y-axis \([0,1]\)
4. Find \( A(y) \):
   \[ A(y) = A_{\text{big}}(y) - A_{\text{small}}(y) \]
   \[ = \pi [R(y)]^2 - \pi [r(y)]^2 \]
   \[ = \pi \left[ (R(y))^2 - (r(y))^2 \right] \] \((\ast)\)

4a. Eq. of left bndry: \( x=1 \)
4b. Eq. of rt. bndry: \( y = mx + b \) \( y = x-1 \) so \( x = y + 1 \)
   \[ = R(y) = y+1 \quad r(y) = 1 \]

4b. \( A(y) = \pi \left[ (y+1)^2 - 1^2 \right] = \pi \left[ y^2 + 2y + 1 - 1 \right] \]
   \[ = \pi \left[ y^2 + 2y \right] \]

5. \[ V = \int_{y=c}^{y=d} A(y) \, dy = \pi \int_{y=c}^{y=1} (y^2 + 2y) \, dy = \pi \left[ \frac{y^3}{3} + y^2 \right]_{y=c}^{y=1} \]
   \[ = \pi \left( \left[ \frac{1}{3} + 1 \right] - (0+0) \right) = \frac{4\pi}{3} \]

6. The volume is \( \frac{4\pi}{3} \) unit\(^3\).
§ 6.2: SHELL METHOD (p.438).

Sketch.

Here we are re-doing 6.1: p.436: #45, but here we use the "SHELL METHOD".

CLASS ENDS

After Class - continue example:

1. I.D. Δ & Δx
2. Determine Interval (Limits of Integration).
   \[ 1 \leq x \leq 2 \]

4. Now this is DIFFERENT: Here we must find the VOLUME of a SHELL.
   \[ V_K = L \times W \times H = 2\pi r(x) \cdot \Delta x \cdot h(x) = 2\pi r(x)h(x) \Delta x \]  

4a. \( r(x) = x \) \( \Leftrightarrow \) \( r(x) \) is just "how far from the y-axis is the shell".

4b. \( h(x) = \) top function - bottom function
   \[ h(x) = (1) - (x-1) = 2-x \]

5. \( V_K = 2\pi x(2-x)dx \)

6. \[ V \approx \sum_{k=1}^{n} V_k \]
   \[ V = \int_{x=1}^{x=2} 2\pi x(2-x)dx \] cont
cont...

\[
\begin{align*}
&= 2\pi \int_{x=1}^{x=2} (2x - x^2) \, dx = 2\pi \left\{ \frac{x^2}{2} - \frac{x^3}{3} \right\} \bigg|_{x=1}^{x=2} \\
&= 2\pi \left\{ \left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right) \right\} = 2\pi \left\{ 3 - \frac{7}{3} \right\} \\
&= 2\pi \cdot \frac{2}{3} = \frac{4\pi}{3} \\
\end{align*}
\]

\[\boxed{\frac{4\pi}{3}}\]

\[\boxed{\text{The volume is } \frac{4\pi}{3} \text{ units}^3.}\]

Note: Same answer as with the "Washer Method."