I. Review of Yesterday's Additions to After-Class Notes.

II. Recall FTC II: Given a function $y = f(x)$. If $\exists F(x)$ s.t. $F'(x) = f(x)$, then

$$\int_{x=a}^{x=b} f(x) \, dx = F(b) - F(a).$$

III. Evaluate

$$\int_{x=1}^{x=2} e^{x^2} \, dx$$

(Don't Try).

(Sometimes there is no "F"!)


V. §6.3: Lengths of Plane Curves (p. 446).

A. What is the length of the "shoreline?"

B. Length of a Parametrically Defined curve: (p. 446).

(Review Nxt. Pg).
Example \[ \begin{cases} x = \cos(t) \\ y = \sin(t) \end{cases} \quad 0 \leq t < 2\pi \]

What is the graph? Ans: Circle. Center \((0,0)\), \(r=1\).

Parameter? (\(t\))

Look at \(x^2 = (\cos(t))^2 = \cos^2(t)\).

\[
y^2 = \sin^2(t)
\]

So \(x^2 + y^2 = \cos^2(t) + \sin^2(t) = 1\)

\[
x^2 + y^2 = 1 \quad \text{More Traditional Format}
\]

2. Suppose \(x = f(t)\) and \(y = g(t)\) \(t \in [a,b]\).

and suppose \(f, g\) are differentiable on \([a,b]\]

and \(f'\) and \(g'\) are not both equal to zero for any \(t \in [a,b]\) — We call this situation a

Smooth Curve.

\[
\begin{align*}
\Delta x_k & \quad \Delta y_k \\
P_{k-1} & \quad P_k (f(t_k), g(t_k)) \\
P_{k-1} (f(t_{k-1}), g(t_{k-1})) & \quad P_k
\end{align*}
\]

(a) \(L_k = \sqrt{(\Delta x_k)^2 + (\Delta y_k)^2} = \sqrt{[f(t_k) - f(t_{k-1})]^2 + [g(t_k) - g(t_{k-1})]^2}
\)

CLASS ENDS

cont...

\[
= \sqrt{\left(\frac{f(t_k) - f(t_{k-1})}{t_k - t_{k-1}}\right)^2 + \left(\frac{g(t_k) - g(t_{k-1})}{t_k - t_{k-1}}\right)^2} \cdot (t_k - t_{k-1})
\]
Let } Δt_k = t_k - t_{k-1} \text{.}

首次函数的平方差和次函数的平方差的和的平方根与时间差的乘积:

\[ \sqrt{\frac{(f(t_k) - f(t_{k-1}))^2 + (g(t_k) - g(t_{k-1}))^2}{(Δt_k)^2}} \cdot Δt_k \]

根据 MVT (Mean Value Theorem) 存在 } t_k^* \text{ 和 } t_k^{**} \text{，使得在区间 } (t_{k-1}, t_k) \text{ 中，有 } \frac{f(t_k) - f(t_{k-1})}{t_k - t_{k-1}} = f'(t_k^*) \text{。}

and \quad \frac{g(t_k) - g(t_{k-1})}{t_k - t_{k-1}} = g'(t_k^{**}) \text{。}

\[ \sqrt{\left[f'(t_k^*)\right]^2 + \left[g'(t_k^{**})\right]^2} \cdot Δt_k \]

Thus \quad L \approx \sum_{k=1}^{n} L_k = \sum_{k=1}^{n} \sqrt{\left[f'(t_k^*)\right]^2 + \left[g'(t_k^{**})\right]^2} \cdot (Δt_k)

\[ L = \lim_{n \to \infty} \sum_{k=1}^{n} \sqrt{\left[f'(t_k^*)\right]^2 + \left[g'(t_k^{**})\right]^2} \cdot (Δt_k) \]

\[ L = \int_{t=a}^{t=b} \sqrt{\left[f'(t)\right]^2 + \left[g'(t)\right]^2} \, dt \quad \text{IF the limit exists!} \]

What happens is that as the length of the interval } (t_{k-1}, t_k) \text{ gets smaller-and-smaller, the } t_k^* \text{ and } t_k^{**} \text{, which are both in the interval get washed closer-and-closer, until, in the limiting process, they become synonymous. } \text{。}