Exp: WaterGlass, toothpick, fork, spoon—shows C.M. need not be within the mass.

§ 6.4: Moments & C.M. — Go to (p.455)

A. Wires and thin Rods — Read! (p. 455)

B. If a parallelepiped of "thickness" $\Delta x$ is of uniform density, then we often speak of the "density of the face."

Ex: $\delta(x)$ in units

$$\frac{\text{mass}}{\text{unit}^2}$$

but we understand what is meant.

C. If I have a volume as shown above, with face a rectangle and of constant density, then its C.M. is at the geometric center of the rectangular face:

D. We use these ideas to find the C.M. of many different shapes.

Example — next page
Beginning of Example.

Given

\[ (y - 4) = -(x - 0)^2 \]

Check: \( y = 0 \implies -4 = -x^2 \) \( \implies x = \pm 2. \)

\[ y = 4 - x^2 \]

Thin plate with face the region bounded by

\[ y = 4 - x^2, \quad y = 0, \]

and of uniform density \( \delta(x) = k. \)

Find the C.M.

Our job: \( M, M_x \) (moment w/ respect to x-axis), \( M_y \) (moment w/ respect to y-axis), \( \bar{x} \) (x-coord. of C.M.), \( \bar{y} \) (y-coord. of C.M.)

Report C.M. \( (\bar{x}, \bar{y}) = \) ?

Method: Steps. \( \) (Uniform Density).

Basic Element of Area

Conclusion: \( \bar{y} = \frac{1}{2} f(\bar{x}) \)
See "Program" in Ex 3, p. 459.

1. Find c.m. of element of area.
2. Length of EofA.
3. Width of EofA.
4. Find \( dA \) (element of area).
5. "" mass.
6. dist c.m. of EofA to y-axis \( \bar{x} = x \)
7. moment \( \bar{x} \delta dA \).
8. Integrate.

END of CLASS