Today's Activities: § 6.5: Areas of Surfaces of Revolution & Pappus's Theorems. (p. 465)

A. Recall "Formula" for Length of a Curve.

If \( y = f(x) \), \( x \in [a, b] \), \( f \) describes a smooth curve, then the LENGTH of the curve, (call it \( L \)) is given by

\[
L = \int_{x=a}^{x=b} \sqrt{1 + [f'(x)]^2} \, dx
\]

B. Look at a "piece" of a curve (p. 467)

So the SURFACE AREA formula is:

\[
S = \int_{x=a}^{x=b} 2\pi f(x) \sqrt{1 + [f'(x)]^2} \, dx
\]
If we are rotating about the \( y \)-axis, the formula becomes
\[
S_c = \int_{y=c}^{y=d} 2\pi x \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy
\]
\[
= \int_{y=c}^{y=d} 2\pi g(y) \sqrt{1 + [g'(y)]^2} \, dy
\]
where \( g'(y) = \frac{dx}{dy} \)

**Pappus Theorem** pp. 472-473 (Skip).

\( \text{§ 6.5: p. 474: #10. Find the lateral surface area of the cone generated by revolving the line segment} \)
\[
y = \frac{1}{2} x \quad x \in [0, 4] \text{ abt the } y \text{-axis.}
\]
\[
\text{[Check ans } S_c = \frac{1}{2} (\text{base circum.}) \times (\text{slant ht.}) \text{.}]
\]

**Solutions:**

1. 
   \[ y = \frac{1}{2} x \]
   \[ x = 2y \quad \checkmark \]
   \[ y \in [0, 2] \]

2. 
   \[ S_c = \int_{y=0}^{y=2} 2\pi x \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy = \int_{y=0}^{y=2} 2\pi (2y) \sqrt{1 + [2]^2} \, dy \]
   \[ = 4\pi \sqrt{5} \int_{y=0}^{y=2} y \, dy = \frac{4\pi \sqrt{5}}{2} \left\{ y^2 \right|_{y=0}^{y=2} = 2\pi \sqrt{5}, y = 8\pi \sqrt{5} \text{ units}^2 \]

3. The surface area is \( 8\pi \sqrt{5} \text{ units}^2 \).

4. Check: base circum. is \( 2\pi r = 2\pi 4 = 8\pi \). Slant ht:\n   \[ s.h. = \sqrt{4+16} = 2\sqrt{5} \]
   Formula \( S_c = \frac{1}{2} (\text{base circum.}) \times (s.h.) = \frac{1}{2} (8\pi) 2\sqrt{5} = \frac{8\pi \sqrt{5}}{2} \text{ units}^2 \) \( \checkmark \)