A There is an example of a § 6.4 CM. problem on the web page.

B Pop Quiz (Real) #1: Due 8:10; Integrate
\[ \int_{x=0}^{x=7} x\sqrt{16-x^2} \, dx \]

II NEW! § 6.6: Work (p. 477)

A Simplest Model is Constant Force along a Straight Line.
\[ W = F \cdot d \]

B Variable Force Model. \[ \Delta W = \int F(x) \Delta x \]
\[ W = \int_{x=a}^{x=b} F(x) \, dx \quad (p. 478) \]

C Examples.

1. Hooke's Law \[ \frac{F}{x} = k \quad \boxed{F = kx} \]

2. Example 2 p. 478: Compression of a Spring.

Find the work required to compress a spring from its natural length of 1 ft. to 0.75 ft.
if \( k = 16 \, \text{lb/ft} \)

Solution

\[ F(x) = kx \quad F(x) = 16x \quad x \in [0, \frac{1}{4}] \]
\[ W = \int_{x=0}^{x=b} F(x) \, dx = \int_{x=0}^{x=\frac{1}{4}} 16x \, dx \]
\[ = \text{etc. (finished after class "fact")} \]
\[ = 8 \left. \left[ x^2 \right] \right|_{x=0}^{x=\frac{1}{4}} = 8 \left( \frac{1}{16} - 0 \right) = \frac{1}{2} \, \text{ft-lb} \]

3 Summary: \[ W = \frac{1}{2} \, \text{ft-lb} \]
[3] Stretching a spring. (p. 479)

Look!

[Unstretched]

[Stretched]

[4] Rope in Bucket. (We'll do this tomorrow).

[AC = After Class]

III

I "did" the Quiz (above) at the chalkboard. Here it is:

Exact Ans: \[ \int_{x=0}^{x=7} x \sqrt{16 - x^2} \, dx \]

Solv: Let \( u = 16 - x \), \( du = -dx \), \( dx = -du \), \( x = 16 - u \);

\[ x = 0 \Rightarrow u = 16 \quad \land \quad x = 7 \Rightarrow u = 9 \]

On subs: \[ \int_{x=0}^{x=7} x \sqrt{16 - x^2} \, dx = - \int_{u=16}^{u=9} (16-u) \, \frac{u^{1/2}}{2} \, du \]

\[ = \int_{u=9}^{u=16} (16u^{1/2} - u^{3/2}) \, du = 16 \cdot \frac{2}{3} u^{3/2} - \frac{2}{5} u^{5/2} \bigg|_{u=9}^{u=16} \]

\[ = \frac{32}{3} \left( u^{3/2} \right)_{u=9}^{u=16} - \frac{2}{5} \left( u^{5/2} \right)_{u=9}^{u=16} = \frac{32}{3} (64 - 27) - \frac{2}{5} (1024 - 243) \]

\[ = \frac{32}{3} \cdot 37 - \frac{2}{5} \cdot 781 = \frac{1184}{3} - \frac{1562}{5} \]

\[ = \frac{5920 - 4686}{15} = \frac{1234}{15} \]