27 Wed 09.30.09 MAC 2312 - 77 551 17 Students

I Quiz #2.4 \(\int x^5 e^x \, dx\)

A

\[ \int x^5 e^x \, dx = x^5 e^x - \frac{5}{1!} x^4 e^x + \frac{20}{2!} x^3 e^x - \frac{60}{3!} x^2 e^x + \frac{120}{4!} x e^x - \frac{120}{5!} e^x + C \]

Note: 120 = 5! Recall: \( n \in \mathbb{Z}^+ \), \( n! = n(n-1)(n-2)\ldots \cdot 3 \cdot 2 \cdot 1 \)

II Start §8.3: "PARTIAL FRACTIONS," (p. 554)

A The IDEA:

1 Consider: \( \frac{A}{B} + \frac{C}{D} = \frac{AD + BC}{BD} \) ("Putting Together")

2 Also \( \frac{AD + BC}{BD} = \frac{A}{B} + \frac{C}{D} \) ("Taking Apart")

\[ \text{Partial fraction decomposition (PFD)} \]

B Why do it?

1 Consider \( \frac{1}{(x+3)(x-1)} \) Assume \( \frac{A}{x+3} + \frac{B}{x-1} \)

Put Back Together \( \frac{A(x-1) + B(x+3)}{(x+3)(x-1)} \) alg. \( \frac{Ax - A + Bx + 3B}{(x+3)(x-1)} \)

\( = \frac{(A+B)x + (-A+3B)}{(x+3)(x-1)} \)
\[
\frac{1}{(x+3)(x-1)} = \frac{(A+B)x + (-A+3B)}{(x+3)(x-1)}
\]

3. \[1 = (A+B)x + (-A+3B)\]
\[\text{Note: } \frac{A}{B} = \frac{C}{B} \Rightarrow A = C\]

4. Coef. of \(x\) on the LHS must equal the coef. of \(x\) on the RHS.
\[0 = A + B\]
and Const. term on LHS must equal Const. term on RHS.
\[1 = -A + 3B\]

5. The problem becomes: Solve 2 equations in 2 unknowns.
\[A + B = 0 \quad \Rightarrow \quad A + \frac{1}{4} = 0 \Rightarrow A = -\frac{1}{4}\]
\[-A + 3B = 1 \quad \Rightarrow \quad B = \frac{1}{4}\]

\[
\frac{1}{(x+3)(x-1)} = \frac{-1/4}{x+3} + \frac{1/4}{x-1} = \frac{1}{4} \left[ \frac{1}{x-1} - \frac{1}{x+3} \right]
\]

C Application

1. Consider \[\int \frac{dx}{x^2 + 2x - 3} = \int \frac{dx}{(x+3)(x-1)} = \int \frac{1}{4} \left( \frac{1}{x-1} - \frac{1}{x+3} \right) dx\]
\[= \frac{1}{4} \left[ \int \frac{dx}{x-1} - \int \frac{dx}{x+3} \right] = \frac{1}{4} \left[ \ln \left| \frac{x-1}{x+3} \right| \right] + C\]
\[= \frac{1}{4} \ln \left| \frac{x-1}{x+3} \right| + C\]

2. \[\int \frac{dx}{x^2 + 2x - 3} = \frac{1}{4} \ln \left| \frac{x-1}{x+3} \right| + C\]
**Heaviside Cover-Up Method**

\[ \frac{1}{(x+3)(x-1)} = -\frac{1/4}{x+3} + \frac{1/4}{x-1} \quad (\star) \]

To get the numerator over \( x+3 \) on the right—

Say to yourself: What does it take to "wipe out" \( x+3 \)?

Ans. \((-3)\)

Now look at the LHS (left-hand side): With your finger, "Cover up" the \( x+3 \). What you've got left is essentially \( \frac{1}{x-1} \).

Now "plug" the \((-3)\) in for \( x \). You've got \( \frac{1}{(-3)-1} = \frac{1}{-4} = (-\frac{1}{4}) \)

This is the numerator for \( x+3 \): \( \frac{-1/4}{x+3} \)

To get the numerator over \( x-1 \) on the right: The number \(1\) "Wipes-out" \( x-1 \).

Cover-Up \( (x-1) \) on the left. You are looking at \( \frac{1}{x+3} \).

"Plug in" \(1\) for \( x \): \( \frac{1}{1+3} = \frac{1}{4} \). This is the numerator for \( x-1 \) on the right: \( \frac{1/4}{x-1} \)

That's How I got \((\star)\) above!