We discussed briefly Maple 9.5 and the TI-89 calculator.

GoTo Ch10: Conic Sections & Polar Coords. (p. 669)

A. Cone

B. The Circle. \((x-h)^2 + (y-k)^2 = r^2\)

The graph is a circle w/ center @ \((h,k)\) \&

\[ r = \text{radius} \]

C. The Ellipse.

Coordinates will be justified after.

1. The Std Form.

\[ \overline{F_1P} + \overline{PF_2} = 2a \]

\(P(x,y)\) is any point on the ellipse.

\[ d_1 = \overline{F_1P} = \sqrt{(-c-x)^2 + (0-y)^2} \]

\[ d_2 = \overline{PF_2} = \sqrt{(x-c)^2 + (y-0)^2} \]

\[ \therefore \sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a \]
\[
\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}
\]

"Separate the radicals first"

\[
(A-B)^2 = A^2 - 2AB + B^2
\]

\[
\text{Sq. b.s.} \quad \text{Then square both sides.}
\]

\[
(x+c)^2 + y^2 = (2a)^2 - 2(2a) \sqrt{(x-c)^2 + y^2} + [(x-c)^2 + y^2]
\]

\[
(x+c)^2 + y^2 = 4a^2 - 4a \sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2
\]

\[
x^2 + 2cx + c^2 = 4a^2 - 4a \sqrt{(x-c)^2 + y^2} + x^2 - 2cx + c^2
\]

E.o.c.

END of CLASS

... continued after class

\[
\therefore 4a \sqrt{(x-c)^2 + y^2} = 4a^2 - 4cx
\]

\[
a \sqrt{(x-c)^2 + y^2} = a^2 - cx
\]

\[
\therefore a^2[(x-c)^2 + y^2] = (a^2 - cx)^2 = a^4 - 2a^2(cx) + c^2x^2
\]

\[
a^2(x^2 - 2cx + c^2 + y^2) = a^4 - 2a^2cx + c^2x^2
\]

So \[a^2x^2 - 2a^2cx + a^2c^2 + a^2y^2 = a^4 - 2a^2cx + c^2x^2\]

Now \[a^2x^2 + a^2y^2 = a^4 + c^2x^2 - a^2c^2\]

\[a^2x^2 - c^2x^2 + a^2y^2 = a^4 - a^2c^2\]

\[(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2) \quad (\star)\]

Woops

Move the \(a^2\) term to the LHS!

So now I've got this equation where I want it!
First, to "simplify" my writing, I'm going to define
\[ b^2 \equiv a^2 - c^2 \]
so
\[ a^2 = b^2 + c^2 \] (***)
and replace \( a^2 - c^2 \) in eqn. (3)
\[ b^2x^2 + a^2y^2 = a^2b^2 \] (***)
Now divide b.s. of (****) by \( a^2b^2 \):
\[
\frac{b^2x^2}{a^2b^2} + \frac{a^2y^2}{a^2b^2} = \frac{a^2b^2}{a^2b^2}
\]
So
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.
\] (\( \star \))

This is the STANDARD-FORM equation for a horizontal ellipse centered at the origin.

2. Consequences:
   a. \( y \)-intercepts - Let \( x = 0 \) in (\( \star \))
   \[
   \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad y^2 = b^2 \quad \Rightarrow \quad y = \pm b
   \]
   So the coordinates of the \( y \)-intercepts are \((0, \pm b)\)

b. \[ \begin{array}{c}
\text{By PYT. THM:} \\
?^2 = b^2 + c^2 \\
\text{and by (****) above,} \\
? = a
\end{array} \]
So we have
Now let's examine the $x$-intercepts, which we call the "vertices" of the horizontal ellipse:

Let $y = 0$ in (1)
\[ \frac{x^2}{a^2} = 1 \quad \therefore \quad x^2 = a^2 \quad \therefore \quad x = \pm a \]

So the coordinates of the vertices are $V_1(-a,0)$ and $V_2(a,0)$

Something that I've always thought interesting is

Look at the two instances of "$a$".

Lotus Rectum

and
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a^2 = b^2 + c^2 \]
\[ b^2 = a^2 - c^2 \]
\[ \therefore \quad \frac{c^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \therefore \quad y^2 = b^2 \left(1 - \frac{c^2}{a^2}\right) = \frac{b^2}{a^2} (a^2 - c^2) \]
\[ = \frac{(a^2 - c^2)(a^2 - c^2)}{a^2} = \frac{(a^2 - c^2)^2}{a^2} \]

So
\[ y = \pm \frac{(a^2 - c^2)}{a} = \pm \frac{b^2}{a} \quad \therefore \quad \text{So the point } P(x,y) \text{ is} \]

$P(c, \frac{b^2}{a})$ and the length of the Lotus Rectum $PQ$ is $\frac{2b^2}{a}$