I  
On Calculator:  \[ r_1 = \{1, -1\} \sqrt{2} \sin(3\theta) \]
On Paper:  \[ r_1 = \pm \sqrt{2} \sin 3\theta \]
\[ r_1^2 = 2 \sin 3\theta \]  "missing" some pieces of graph!? 

II  
\[ \S 10.7 : \text{p. 714 : #3} \]
Find the area of one "leaf" of  \( r = \cos 2\theta \)

Solu:  \[ 3 \]

Find Area of "Half-a-Leaf" and double it.

II  
\( A_{\text{leaf}} = ? \)  
What is the formula?  See III below for formula!

III  
Formula for Area:

2  
Formula for a Circular Sector:  \[ A = \frac{1}{2} r^2 \theta \]  \( \theta \) in radians

3  
In Fig. 1.
\[ A_k = \frac{1}{2} r_k^2 \Delta \theta_k \]  where  \( \Delta \theta_k = \theta_k - \theta_{k-1} \)

4  
\[ A \approx \sum_{k=1}^{n} A_k = \sum_{k=1}^{n} \frac{1}{2} r_k^2 \Delta \theta_k \]  
Take limit as  \( n \to \infty \), we see  (if limit exists)  \[ A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \, d\theta = \int_{\theta_{\alpha}}^{\theta_{\beta}} \frac{1}{2} [f(\theta)]^2 \, d\theta \]  

B  
Now continue II w/ this formula.
Continuation: 

\[ A_{\text{leaf}} = 2 \int_{\Theta = 0}^{\Theta = \pi/4} \frac{1}{2} \left[ \cos(2\Theta) \right]^2 d\Theta \]

\[ = \frac{1}{2} \int_{\Theta = 0}^{\Theta = \pi/4} (1 + \cos(4\Theta)) d\Theta \]

\[ = \frac{1}{2} \left\{ \Theta + \frac{1}{4} \sin(4\Theta) \right\} \bigg|_{\Theta = 0}^{\Theta = \pi/4} \]

\[ = \frac{1}{2} \left\{ \left( \frac{\pi}{4} + 0 \right) - (0) \right\} = \frac{\pi}{8} \text{ units}^2 \]

Length of Polar Curve. (p. 712).

\[ L = \int_{\Theta = \alpha}^{\Theta = \beta} \sqrt{\left( \frac{dx}{d\Theta} \right)^2 + \left( \frac{dy}{d\Theta} \right)^2} \ d\Theta \]

Known formula

\[ T: \begin{cases} x = f(\Theta) \cos \Theta \\ y = f(\Theta) \sin \Theta \end{cases} \]

where \( r = f(\Theta) \)

and \( \alpha \leq \Theta \leq \beta \).

\[ \frac{dx}{d\Theta} = f(\Theta) (-\sin \Theta) + f'(\Theta) \cos \Theta = f(\Theta) \cos \Theta - f(\Theta) \sin \Theta \]

\[ (\frac{dx}{d\Theta})^2 = \left( f'(\Theta) \right)^2 \cos^2 \Theta - 2 f'(\Theta) f(\Theta) \sin \Theta \cos \Theta + (f(\Theta))^2 \sin^2 \Theta \]

and \[ \frac{dy}{d\Theta} = f(\Theta) \cos \Theta + f'(\Theta) \sin \Theta \]

\[ (\frac{dy}{d\Theta})^2 = \left( f'(\Theta) \right)^2 \cos^2 \Theta + 2 f'(\Theta) f(\Theta) \sin \Theta \cos \Theta + (f'(\Theta))^2 \sin^2 \Theta \]
So \( (dx)^2 + (dy)^2 \)

\[\begin{align*}
&= (f'(\theta))^2 \cos^2 \theta - 2f'(\theta)f(\theta) \sin \theta \cos \theta + (f(\theta))^2 \sin^2 \theta \\
&\quad + (f'(\theta))^2 \sin^2 \theta + 2f'(\theta)f(\theta) \sin \theta \cos \theta + (f(\theta))^2 \cos^2 \theta \\
&= (f'(\theta))^2 \cos^2 \theta + (f'(\theta))^2 \sin^2 \theta + (f(\theta))^2 \cos^2 \theta + (f(\theta))^2 \sin^2 \theta \\
&= (f'(\theta))^2 + (f(\theta))^2 = (f(\theta))^2 + (f'(\theta))^2
\end{align*}\]

which is also equal to \( r^2 + (\frac{dr}{d\theta})^2 \).

**D** So the "formula" for the length of a polar curve is

\[L = \int_{\theta = \alpha}^{\theta = \beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta\]

**IF** \( r = f(\theta) \) has a continuous 1st derivative for \( \alpha \leq \theta \leq \beta \) and

**IF** the point \( P(r, \theta) \) traces the curve \( r = f(\theta) \) exactly once as \( \theta \) runs from \( \alpha \) to \( \beta \).