I returned Test #3 A & B, but not notebooks.

II From 11.3 - The Integral Test (p. 756).

A Review. We Did

1. The Harmonic Series Diverges (p. 756)
\[ \sum_{n=1}^{\infty} \frac{1}{n} \rightarrow \text{Harm. Series.} \]

2. The series (a member of the so-called "p-series")
\[ \sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow \text{converges.} \]

The Seq. of Partial Sums \( \{S_n\} = \left\{ \sum_{i=1}^{n} \frac{1}{n^2} \right\} \)
is bdd. (bounded) above by 2.

3. The Important Thing Here is that \( \sum_{n=1}^{\infty} \frac{1}{n} \rightarrow \text{Diverges} \)
and \( \sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow \text{converges.} \)

B Thm 9 The Integral Test (p. 757)

1. Given a seq. \( \{a_n\} \). S \exists a function \( y = f(x) \)
   0 continuous on some interval \( \Omega [N, \infty) \), decreasing on \( [N, \infty) \)
   2 pos. \( [N, \infty) \) \( (N \in \mathbb{Z}^+) \), and \( f(n) = a_n \) \( \forall n \geq N \).

   THEN \( \sum_{n=1}^{\infty} a_n \) and \( \int_{x=1}^{\infty} f(x)dx \) both converge.

2. (If they DO converge, it is not necessarily to the same limit)

3. This may provide us w/ a test for series convergence, because
   sometimes it is "easier" to see if the integral converges.
2. \[ \lim_{n \to \infty} \frac{a_n}{n^2} \]

"Tail" must be "nice" / True for Sequences and Series.

3. For Examples:

[a] \[ \sum_{n=1}^{\infty} a_n = 5 \]  \[ \sum_{n=3}^{\infty} a_{n-2} = 5 \]  is the same series 

and \[ \sum_{n=1}^{\infty} b_n = 3 + 7 + a_1 + a_2 + \ldots \]

\[ = \sum_{n=1}^{\infty} b_n = 3 + 7 + a_1 + a_2 + \ldots \]

\[ = 10 + 5 = 15 \]

[b] If \[ \sum_{n=1}^{\infty} a_n = L \] then \[ A + B + \ldots + K + \sum_{n=1}^{\infty} a_n = A + B + \ldots + K + L \]

---

after class

[c] We know (?) that \[ \sum_{n=1}^{\infty} 2 \left( \frac{1}{3} \right)^{n-1} = 3 \]

If it is a GS w/ \( a = 2 \) and \( r = \frac{1}{3} \) and \[ \sum_{n=1}^{\infty} 2 \left( \frac{1}{3} \right)^{n-1} = \frac{a}{1-r} = \frac{2}{3/3} = 3 \]

Thus the series \[ \frac{1}{2} + \frac{1}{8} + 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots \]

\[ = \left( \frac{1}{2} + \frac{1}{8} \right) + \left( 2 + \frac{2}{3} + \frac{2}{9} + \ldots \right) = \frac{5}{8} + \sum_{n=1}^{\infty} 2 \left( \frac{1}{3} \right)^{n-1} = \frac{5}{8} + 3 \]

\[ = \frac{29}{8} \]

\[ \frac{1}{2} + \frac{1}{8} + 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \ldots = \frac{29}{8} \]