Section 11.2: Permutations finished  (Objectives 1 - 4)
(Permutations & Combinations)
We got to the middle of Page 2 on Tue. 11/10/09 - djj

Permutation: an ordered arrangement of a set of items (no repetition allowed).

ex. In how many ways can 6 children line up to walk to the library?
\[ 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720 \]

“Count-down”

What if the teacher's pet, Carolyn, is always the leader?
\[ 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \]

ex. How many ways are there to seat 7 students in the 7 desks of row 1?
\[ 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 7! = 5040 \]

Write this product as 7! Use calculator key \[ \text{x!} \].
**Factorial:** \( n! = n(n-1)(n-2) \ldots (3)(2)(1) \). It determines the number of permutations of \( n \) distinct items.

Note: \( 0! = 1 \)

\[
0! = 1 \quad \text{BY DEF.}
\]

ex. I want to arrange 4 photos in a row on my desk. How many arrangements are possible?

\[
\begin{array}{cccc}
4 & 3 & 2 & 1 \\
\end{array}
\]

\[= \boxed{24}\]

ex. Evaluate:

\[
\frac{12!}{10!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \ldots 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8 \ldots 3 \cdot 2 \cdot 1} = \boxed{132}
\]

\[
\frac{8!}{(8-5)!} = \frac{8 \cdot 7 \cdot 6 \ldots 1}{3 \cdot 2 \cdot 1} = \boxed{6720}
\]

---

GOT TO HERE TUE 11/10/09 === Start Thu. 11/12 ==

Permutations can use **all** the elements of a given set or **only a certain number** of them.

ex. I have 24 photos of my trip to Yellowstone National Park. I only have room for 3 frames on my desk. In how many ways can the photos be selected and arranged in row on my desk?

\[
\begin{array}{c}
24 \\
23 \\
22 \\
\end{array}
\]

\[= \boxed{12144}\]
This formula counts the number of ways to choose (without replacement) and then arrange, r items out of n distinct items.

\[ n \text{P}_r = \frac{n!}{(n-r)!} \]

ex. Use the permutation formula to do the previous "Yellowstone" example.

\[ 7 \text{P}_7 = 7! = 5040 \]

Note: \( n \text{P}_n = n! \).

ex. Use the permutation formula to do the previous "7 students seated in 7 desks" example.

A television network must schedule 6 half-hour programs during prime-time (7 - 10 pm). If there are 9 programs to choose from, how many program schedules are possible?

\[ 9 \text{P}_6 = 60480 \]

\[ 9 \text{P}_6 = \frac{9!}{(9-6)!} = \frac{9!}{3!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4}{3!} = 60480 \]
ex. How many permutations are there of the letters in "CAT"?
\[ CAT \quad TAC \quad TCA \quad ACT \quad ATC \]
\[ _3 P_3 = 3! = 6 \]

ex. How many permutations are there of the letters in "ALL"?

### Permutations of Duplicate Items

The number of permutations of \( n \) items, where \( p \) items are identical, \( q \) items are identical, \( r \) items are identical, etc. is given by

\[ \frac{n!}{p!q!r!...} \]

ex. How many distinct ways can the letters of the word "coffeecake" be arranged? (It's a word, so order is imp.)
\[ n = 10 \quad \frac{10!}{2!2!3!} = \frac{10!}{24} = 151200 \]

ex. How many distinct ways can the letters of the word "Tallahassee" be arranged?
\[ \frac{11!}{3!2!2!2!} = \frac{11!}{48} = 831600 \]

See text ex. 1 - 6.
In how many ways can one arrange the letters of the word "ALL"?

\[
\frac{n!}{P!} = \frac{3!}{2!} = \frac{6}{2} = 3
\]

How many distinct words can be made from the letters of "ALL"?

3