Section 11.5: Probability with the FCP, Permutations, and Combinations

(Objectives 1 - 2)

We started this Section on Thurs. 11/19/09 and we finished Tue. 11/24/09

Recall the following counting problems from 11.3:

ex. How many 6-number picks are there in today's Lotto game (choose numbers from 1-53)?

\[ \binom{53}{6} = \frac{53!}{(53-6)! \cdot 6!} = 22,957,480 \]

If you buy 1 Lotto ticket, what is the probability of winning?

\[ P(\text{winning}) = \frac{1}{22,957,480} \approx 0.0000000436 \]

With 10 tickets, \[ P(\text{winning}) = \frac{10}{22,957,480} \approx 0.000000436 \]
ex. How many three-member committees consisting of all women can be formed from a group of 4 men and 5 women?

\[ \binom{5}{3} = \frac{5!}{(5-3)! \ 3!} = \binom{10}{6} \]

If 3 members are randomly selected from this group of 4 men and 5 women, what is the probability the selected group consists of all women?

\[ P(\text{all women}) = \frac{\# \text{ of all-women groups}}{\text{total \# of groups}} = \frac{\binom{5}{3}}{\binom{9}{3}} = \frac{\binom{10}{6}}{\binom{12}{6}} = \frac{5}{42} \]

======== We got to here on Thurs. 11/19/2009 ========
Hw. #2: Three men and three women line up at a checkout counter in a store.

a) In how many ways can they line up?

\[
\text{Grand Total (Bottom)} = \frac{6! \cdot 3! \cdot 2!}{1!} = 6! = 720
\]

\[
\text{Sample Space (S)} = 6! \cdot 6! = 720 \cdot 720 = 518,400
\]

b) In how many ways can they line up if the first person in line is a woman, and then the line alternates by gender?

\[
3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 36
\]

\[
3 \cdot 3 \cdot 3 = 3! \cdot 3! = 36
\]

\[
3 \cdot 3 \cdot 3 = 3! \cdot 3! = 36
\]

\[
= 6 \cdot 6 = 36
\]

\[
\text{EVENT} = \frac{36}{720} = \frac{1}{20}
\]

c) Find the probability that the first person in line is a woman and the line alternates by gender?
Hw.# 4: Seven performers, A, B, C, D, E, F, and G, are to appear in a fund-raiser. The order of performance is determined by random selection. Find the probability that

a) D will perform first.

\[ P(D) = \frac{5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{5} \]

b) E will perform sixth and B will perform last.

\[ P(E_6 B_{last}) = \frac{5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{42} \]

c) They will perform in the following order: C, D, B, A, G, F, E.

\[ P(C, D, B, A, G, F, E) = \frac{1}{5040} \]

d) F or G will perform first.

\[ P(F \text{ or } G) = P(F) + P(G) = \frac{6!}{7!} + \frac{6!}{7!} = 2 \times \frac{1}{7} = \frac{2}{7} \]
Hw.# 10: A **committee of 5** people is to be formed from **six** lawyers and **seven** teachers. Find the **probability** that

a) all are lawyers.

\[
P(\text{All L}) = \frac{\binom{6}{5}}{\binom{13}{5}} = \frac{6}{1287}
\]

\[
P = \frac{\binom{6}{5}}{\binom{13}{5}} = \frac{6}{1287}
\]

b) none are lawyers. (read this as "all are teachers.")

\[
P(\text{All T}) = \frac{\binom{7}{5}}{\binom{13}{5}} = \frac{7}{429}
\]

Hw.# 12: A parent-teacher committee consisting of **four** people is to be selected from **fifteen** parents and **five** teachers. Find the probability of selecting **two** parents and **two** teachers.

See text ex. 1 - 3.