Section 2.3: Venn Diagrams and Set Operations

Objectives 1-10

Universal Set: set that contains all the elements under discussion, symbolized by $U$.

Venn Diagram: used to visualize set relationships, 1st used in 1881 by John Venn (1834 - 1923) in Symbolic Logic. This English mathematician resigned from the priesthood after 24 years so he could devote his efforts to the study of logic!

$U$: set of TCC students.

$A$: set of TCC students enrolled in MGF 1106.
Venn Diagrams Relating 2 Sets

- Disjoint: no common elements
- Proper Subset: \( A \subset B \)
- Equal: Elements in Common

Set Operations

Complement of \( A \): elements in \( U \), but not in \( A \), denoted \( A' \).

\[ A' = \{ x | x \in U \text{ and } x \notin A \} \]

ex. \( U = \{1, 2, 3, 4, 5\} \)
\( A = \{2, 4\} \)
\( A' = \{1, 3, 5\} \)
Hw.#18: Let \( U = \{1, 2, 3, 4, 5, 6, 7, 8\} \) and \( A = \{2, 3, 6, 8\} \).

Find \( A' \) and place the elements in the proper regions.

Intersection of \( A \) and \( B \):

- Elements common to both \( A \) and \( B \), denoted \( A \cap B \).
- \( A \cap B = \{x | x \in A \text{ and } x \in B\} \)

Union of \( A \) and \( B \):

- Elements that are members of \( A \) or \( B \) (or both), denoted \( A \cup B \).
- \( A \cup B = \{x | x \in A \text{ or } x \in B\} \)

ex. Let \( U = \{1, 2, 3, 4, 5, 6, 7, 8\} \)

\( A = \{2, 3, 6, 8\} \quad B = \{6, 7, 8\} \)

Find \( A \cap B = \{6, 8\} \) \quad Find \( A \cup B = \{2, 3, 6, 7, 8\} \)
ex. \( U = \{1, 2, 3, 4, 5, 6, 7, 8\} \quad A = \{1, 2, 3\}\)
\( B = \{2, 3, 4, 5\} \quad C = \{5, 6, 7\} \quad D = \{\}\)

\[ B' = \{1, 6, 7, 8\} \quad A' = \{4, 5, 6, 7, 8\} \]

\( A \cap B = \{2, 3\} \quad B' = \{1, 6, 7, 8\} \)

\( B \cup C = \{2, 3, 4, 5, 6, 7\} \quad C' = \{1, 2, 3, 4, 8\} \)

\[ A \cap C = \emptyset \quad \text{or} \quad \{\} \]

\( A' \cap C = \{5, 6, 7\} = C \)

\[ B' \cup C' = \{1, 2, 3, 4, 6, 7, 8\} \]

\( (B \cup C)' = \{1, 8\} \) \text{ illustrates 1 of DeMorgan's Laws \( (B \cup C)' = B' \cap C' \)}

\( B' \cap C' = \{1, 8\} \)

\( B \cup D \quad \text{Other} \quad (B \cap C)' = B' \cup C' \)

\[ A \cap D \]

Note: For any set \( A \), \( A \cap \emptyset = \emptyset \) and \( A \cup \emptyset = A \).
Hw. #76: List the elements of the set in roster form:

\[ A' \cap B = \{2, 5, 6\}\]

\[ A' = \{2, 5, 6, 8, 9\}\]

\[ B = \{2, 3, 5, 6, 7\}\]

\[ A \cap C = \{3, 5\}\]

Hw. #104: Let

\[ U = \{x \mid x \in N \text{ and } x < 9\}\]

\[ U = \{1, 2, 3, 4, 5, 6, 7, 8\}\]

\[ A = \{x \mid x \text{ is an odd natural number and } x < 9\}\]

\[ A = \{1, 3, 5, 7\}\]

\[ B = \{x \mid x \text{ is an even natural number and } x < 9\}\]

\[ B = \{2, 4, 6, 8\}\]

\[ C = \{x \mid x \in N \text{ and } 1 < x < 6\}\]

\[ C = \{2, 3, 4, 5\}\]

Find \( (A \cap C)' \):

\[ \{1, 2, 4, 6, 7, 8\}\]
Hw. #109 - #116:

A math tutor working with a small group of student asked each student when he or she had studied for class the previous weekend. Their responses are shown in the Venn diagram.

Use the Venn diagram to list the elements of each set in roster form.

#110. The set of students who studied Sunday

#111. The set of students who studied Saturday or Sunday

\[ \{\text{Ashley, Mike, Josh, Emily, Hanna, Ethan}\} \]

#114. The set of students who studied Sunday and not Saturday

\[ \{\text{Emily, Hanna, Ethan}\} \]

Notice: \( n(\text{studied Saturday}) = 3, \quad n(\text{studied Sunday}) = 5, \quad \text{but} \quad n(\text{studied Saturday or Sunday}) = 6 \).
Hw.#94: Set A contains 30 elements, set B contains 18 elements, and 5 elements are common to sets A and B. How many elements are in $A \cup B$?

See text ex.1 - 8.