Section 3.6: Arguments and Truth Tables
(Objectives 1-2)

DEDUCTIVE ARGUMENTS

Argument: consists of given statements called premises and a conclusion.

Valid Argument: conclusion must follow from the given set of premises.

Invalid Argument (or fallacy): conclusion doesn't necessarily follow from given premises.

To determine if an argument is valid, write it in symbols and analyze the symbol pattern.

ex: If I am sleeping, then I am breathing.  
    p → q  
    I am asleep.  
    p  
    ∴ I am breathing.  
    ∴ q

The argument is valid if the conjunction of the premises implies the conclusion for all cases. (TAUTOLOGY)
Valid Argument: conclusion **must** follow from the given set of premises.

**DEDUCTIVE**

Invalid Argument (or fallacy): conclusion doesn't necessarily follow from given premises.

<table>
<thead>
<tr>
<th>FORM (STRUCTURE)</th>
<th>CONTENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Valid</td>
<td>TRUTH VALUES of Premise(s) and Conclusion.</td>
</tr>
<tr>
<td>2. Invalid</td>
<td></td>
</tr>
</tbody>
</table>
Construct a truth table for \((p \rightarrow q) \land p \rightarrow q\).

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p \rightarrow q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

This is a valid argument!

Please see the next page for the entire Truth Table.

The truth of the conclusion does not determine validity. The form of the argument determines validity. In a valid argument, if the premises are true, the conclusion must also be true. A valid argument with true premises is called a sound argument.

ex. If you live in Miami, then you wear a heavy coat every day.
   You live in Miami.
   \(\therefore\) You wear a heavy coat every day.

This is a valid argument. However, the conclusion is not true since the premises are not both true.
Construct a truth table for \([(p \rightarrow q) \land p] \rightarrow q\).

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p\rightarrow q</th>
<th>(p\rightarrow q) \land p</th>
<th>[(p\rightarrow q) \land p] \rightarrow q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
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<td>F</td>
<td>F</td>
<td>T</td>
<td></td>
<td>T</td>
</tr>
</tbody>
</table>

1. 2  3  4

Tautology Arg. is VALID in form.
If an English argument translates into one of these forms, you can immediately determine validity without using truth tables.

<table>
<thead>
<tr>
<th>Direct Reasoning</th>
<th>Contrapositive Reasoning</th>
<th>Disjunctive Reasoning</th>
<th>Transitive Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \rightarrow q )</td>
<td>( \neg q )</td>
<td>( p \lor q )</td>
<td>( q \rightarrow r )</td>
</tr>
<tr>
<td>( p )</td>
<td>( \therefore \neg p )</td>
<td>( \neg p )</td>
<td>( \therefore p )</td>
</tr>
<tr>
<td>( \therefore q )</td>
<td>( \therefore \neg q )</td>
<td>( \therefore \neg r \lor \neg p )</td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{VALID} \]

\begin{align*}
\text{Fallacy of the Converse} & \quad \text{Fallacy of the Inverse} & \quad \text{Misuse of Disjunctive Reasoning} & \quad \text{Misuse of Transitive Reasoning} \\
\text{Invalid} & \quad \text{Invalid} & \quad \text{Invalid} & \quad \text{Invalid} \\
\therefore p & \quad \therefore \neg q & \quad \therefore \neg q & \quad \therefore \neg p & \quad \therefore \neg p \\
\therefore \neg p & \quad \therefore q & \quad \therefore q & \quad \therefore r & \quad \therefore \neg r \\
\end{align*}

\[ \text{INVALID} \]
Valid Form

ex. If I'm sleeping, then I'm breathing.  \( p \rightarrow q \)
   I'm sleeping.  \( p \)
   \( \therefore \) I'm breathing.  \( \therefore q \)

EX. If I'm sleeping, then I'm breathing.  \( p \rightarrow q \)
   I'm not breathing.  \( \sim q \)
   \( \therefore \) I'm not sleeping.  \( \therefore \sim p \)

ex. If I am sick, then I stay in bed.  \( p \rightarrow q \)
   If I stay in bed, then I watch TV.  \( q \rightarrow r \)
   \( \therefore \) If I am sick, then I watch TV.  \( \therefore p \rightarrow r \)

ex. I will go to Publix Monday or Tuesday.  \( p \lor q \)
   I did not go to Publix Monday.  \( \sim p \)
   \( \therefore \) I went to Publix Tuesday.  \( \therefore q \)
**Invalid**

ex. If I'm sleeping, then I'm breathing. \( p \rightarrow q \)
   I'm not sleeping. \( \sim p \)
   \( \therefore \) I'm not breathing. \( \therefore \sim q \)

ex. If I'm sleeping, then I'm breathing. \( p \rightarrow q \)
   I'm breathing. \( q \)
   \( \therefore \) I'm sleeping. \( \therefore p \)

ex. I will go to Publix Monday or Tuesday. \( p \land q \)
   I went to Publix Monday. \( p \)
   \( \therefore \) I did not go to Publix Tuesday. \( \therefore \sim q \)

ex. If I am sick, then I stay in bed. \( p \rightarrow q \)
   If I stay in bed, then I watch TV. \( q \rightarrow r \)
   \( \therefore \) If I watch TV, then I am sick. \( \therefore r \rightarrow p \)
**Drawing Logical or Valid Conclusions**

Draw a valid conclusion from the given premises by translating into symbolic form and using valid patterns.

- If Frank buys a car, then he can drive home. \( p \rightarrow q \)
- Frank cannot drive home. \( \sim q \)

Therefore,

- If you study, then you will pass this class. \( p \rightarrow q \)
- If you pass this class, then you can graduate. \( q \rightarrow r \)

Therefore,
If all stores are closed, then no one can shop.  \( p \rightarrow q \)
Some one can shop. \( \sim q \)

Therefore,

If you work, then you can pay your bills.  \( p \rightarrow q \)
If you can pay your bills, then your credit is good.  \( q \rightarrow r \)
Your credit is not good.  \( \sim r \)

Therefore,

See text examples 1 - 6. Use Handout on Alternative Truth Table Method for Symbolic Arguments.