Section 3.6: Arguments and Truth Tables

(Objectives 1-2) FINISHED IN CLASS TUE. 11.03.09

**DEDUCTIVE ARGUMENTS**

**Argument**: consists of given statements called premises and a conclusion.

**Valid Argument**: conclusion must follow from the given set of premises.

**Invalid Argument** (or fallacy): conclusion doesn't necessarily follow from given premises.

To determine if an argument is valid, write it in symbols and analyze the symbol pattern.

**ex.** If I am sleeping, then I am breathing. p → q
I am asleep. p
∴ I am breathing. q

The argument is valid if the conjunction of the premises implies the conclusion for all cases. (TAUTOLOGY)
**Valid Argument**: conclusion **must** follow from the given set of premises.

**Invalid Argument** (or fallacy): conclusion doesn't necessarily follow from given premises.

<table>
<thead>
<tr>
<th>Form (Structure)</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Valid</td>
<td>Truth values of premise(s) and conclusion.</td>
</tr>
<tr>
<td>2. Invalid</td>
<td></td>
</tr>
</tbody>
</table>
Construct a truth table for \((p \rightarrow q) \land p \rightarrow q\). This is a valid argument!

Please see the next page for the entire Truth Table.

The truth of the conclusion does not determine validity. The form of the argument determines validity. In a valid argument, if the premises are true, the conclusion must also be true. A valid argument with true premises is called a sound argument.

ex. If you live in Miami, then you wear a heavy coat every day. You live in Miami. 
\[\therefore\] You wear a heavy coat every day.

This is a valid argument. However, the conclusion is not true since the premises are not both true.
Construct a truth table for \([(p \rightarrow q) \land p] \rightarrow q\).

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>(p \rightarrow q)</th>
<th>((p \rightarrow q) \land p)</th>
<th>([(p \rightarrow q) \land p] \rightarrow q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
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<tr>
<td>T</td>
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<td>T</td>
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<td>T</td>
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</tbody>
</table>

\[\begin{align*}
1 & 2 & 3 & 4 & 5 \\
\text{T} & \text{F} & \text{T} & \text{T} & \text{T} \\
\text{F} & \text{T} & \text{T} & \text{T} & \text{T} \\
\text{F} & \text{T} & \text{T} & \text{T} & \text{T} \\
\text{F} & \text{T} & \text{T} & \text{T} & \text{T} \\
\end{align*}\]

Tautology: Arg. is VALID in form.
If it rains, Jones gets wet.
If Jones gets wet, then I don't wear it.

The Valid Form:

\[
\begin{align*}
& p \rightarrow q \\
& q \rightarrow r \\
\hline
\therefore & p \rightarrow r
\end{align*}
\]
If an English argument translates into one of these forms, you can immediately determine validity without using truth tables.

**VALID**

**Direct Reasoning**

\[ p \rightarrow q \\
\hline
p \\
\hline
: \quad q \]

**Contrapositive Reasoning**

\[ p \rightarrow q \\
\hline
\sim q \\
\hline
: \quad \sim p \]

**Disjunction Reasoning**

\[ p \lor q \\
\hline
\sim p \\
\hline
: \quad \sim q \\
\hline
: \quad q \\
\hline
: \quad p \]

**Transitive Reasoning**

\[ p \rightarrow q \\
\hline
q \rightarrow r \\
\hline
: \quad \sim r \rightarrow \sim p \]

1. TWO alternatives. (2) Rule out one.

**INVALID**

**Fallacy of the Converse**

\[ p \rightarrow q \\
\hline
q \\
\hline
: \quad p \]

**Fallacy of the Inverse**

\[ p \rightarrow q \\
\hline
\sim p \\
\hline
: \quad \sim q \]

**Misuse of Disjunctive Reasoning**

\[ p \lor q \\
\hline
p \\
\hline
: \quad \sim q \]

\[ p \lor q \\
\hline
q \\
\hline
: \quad \sim p \]

**Misuse of Transitive Reasoning**

\[ p \rightarrow q \\
\hline
q \rightarrow r \\
\hline
: \quad \sim p \rightarrow \sim r \]
Valid

ex. If I'm sleeping, then I'm breathing.
   I'm sleeping.
   \[ p \rightarrow q \]
   \[ p \]
   \[ \therefore q \]

ex. If I'm sleeping, then I'm breathing.
   I'm not breathing.
   \[ \therefore \sim p \]

ex. If I am sick, then I stay in bed.
   If I stay in bed, then I watch TV.
   \[ \therefore p \rightarrow r \]

ex. I will go to Publix Monday or Tuesday.
   I did not go to Publix Monday.
   \[ \therefore q \]
Invalid

ex. If I'm sleeping, then I'm breathing.
   I'm not breathing.
   ∴ I'm not breathing.

ex. If I'm sleeping, then I'm breathing.
   I'm breathing.
   ∴ I'm sleeping.

ex. I will go to Publix Monday or Tuesday.
   I went to Publix Monday.
   ∴ I did not go to Publix Tuesday.

ex. If I am sick, then I stay in bed.
   If I stay in bed, then I watch TV.
   ∴ If I watch TV, then I am sick.
Drawing Logical or Valid Conclusions

Draw a valid conclusion from the given premises by translating into symbolic form and using valid patterns.

If Frank buys a car, then he can drive home.  $p \rightarrow q$
Frank cannot drive home. $\sim q$

Therefore, FRANK DIDN'T BUY A CAR.

If you study, then you will pass this class. $p \rightarrow q$
If you pass this class, then you can graduate. $q \rightarrow r$

Therefore, IF YOU STUDY THEN YOU'LL GRADUATE.
If all stores are closed, then no one can shop. \( p \rightarrow q \)
Some one can shop. \( \sim q \)

Therefore, THE STORES ARE NOT CLOSED.

**IMPORTANT EXAMPLE**

If you work, then you can pay your bills. \( p \rightarrow q \)
If you can pay your bills, then your credit is good. \( q \rightarrow r \)
Your credit is not good. \( \sim r \)

Therefore, \( \sim p \)
\( \text{you didn't work.} \)

See text examples 1 - 6. Use Handout on Alternative Truth Table Method for Symbolic Arguments.