#1. Find the equation in standard form \((Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + G = 0)\) for the sphere with center at \(C(1, 2, 3)\) and which contains the point \(P(-2, 0, 4)\).

#2. A. Find a unit vector that has the same direction as \(\vec{v} = \langle 2, -1, 1 \rangle\).
   B. Find a vector that has the same direction as \(\vec{v} = \langle 2, -1, 1 \rangle\) and has length 9.

#3. If \(\vec{m} = \langle 4, 1, -3 \rangle\) and \(\vec{n} = \langle 0, 3, 2 \rangle\), find the vector projection of \(\vec{m}\) onto \(\vec{n}\).

#4. Find the area of the triangle \(\Delta PQR\) if the vertices have coordinates \(P(1, 1, -1, 1)\), \(Q(2, 8, 5)\), and \(R(5, 0, 1)\). Hint: Use vector product in the solution of this problem. Also, let's say that the "units" are meters. Give exact answer.

#5. Find the direction cosines \((\cos \alpha, \cos \beta, \cos \gamma)\) of the vector \(\vec{v} = \langle 12, 5, 13 \rangle\). Give exact answers, not decimal approximations.

#6. Find parametric and symmetric equations for the line through the points \(P\left(0, \frac{1}{2}, 1\right)\) and \(Q(2, 1, -3)\).

#7. [A]. (2 pts) There are two “Standard Form Equation” forms for the equation of a plane. Write down one of them. [B]. (8 pts) Find the Standard Form Equation for the plane, \(\Pi\), which passes through the point \(P(1, 0, 2)\) and contains the line \(x = 2 + 2t, \ y = -1 + 7t, \ z = -t\).

#8. Find parametric equations for the line of intersection, \(L\), of the planes \(\Pi_1: x + y + z + 1 = 0\) and \(\Pi_2: x - z + 5 = 0\).

[A] (7 pts.) Put the equation into “Table #1 Form” (“Standard Form”), and then [B] (3 pts.) Identify the Quadric Surface involved.

#9. \(-x^2 + 4y^2 - 9z^2 + 36 = 0\)

#10. \(x^2 - z^2 - y = 0\).
BONUS. Given: the point $P(1, -1, 0)$ and the plane $\Pi: 2x + 5y - 4z + 1 = 0$.

Required: [A]. (2 pts.) Is the point $P$ on the plane $\Pi$? (Why?/Why Not?)
[B]. (8 pts.) What is the distance $d(P, \Pi)$ between $P$ and $\Pi$?

#1. (12 pts.) Find a vector equation and parametric equations for the line segment from $P(1, -1, 5)$ to $Q(3, 8, -1)$.

#2. (12 pts.)
A. Change from rectangular to spherical coordinates: $\left(0, \sqrt{3}, 1\right)$.
B. Change from cylindrical to spherical coordinates: $\left(\sqrt{6}, \frac{\pi}{4}, \sqrt{2}\right)$.
C. Change from spherical to cylindrical coordinates: $\left(2\sqrt{2}, \frac{3\pi}{2}, \frac{\pi}{2}\right)$.

#3. (13 pts.) Find the length of the curve $\vec{r}(t) = \left(\sqrt{2}t, e^t, e^{-t}\right)$, $0 \leq t \leq 2$. Give both the exact answer AND the approximate numerical answer to two decimal places. The units are cm.

#4. (13 pts.)
(a) At what point do the curves $\vec{r}_1(t) = \left(t, 2 - t, 2 + t^2\right)$ and $\vec{r}_2(s) = \left(2 - s, s, s^2\right)$ intersect?
(b) Find their angle of intersection correct to the nearest degree.

#5. (12 pts.) Show that this limit does not exist:
$$\lim_{(x, y) \to (0, 0)} \frac{x^2}{x^2 + y^2}$$

#6. (13 pts.) Find the position vector of a particle that has the specified acceleration and the given initial velocity and position.
$$\vec{a}(t) = \left(t\vec{i} + \left(2t^2\right)\vec{j} + \left(\cos(2t)\right)\vec{k}\right), \quad \vec{v}(0) = \vec{i} + \vec{k}, \quad \vec{r}(0) = \vec{j}.$$

#7. (12 pts.)
[A] Find $f_y(1, 5, 3)$ if $f(x, y, z) = \frac{x}{2y + z}$
[B] Find $\frac{\partial z}{\partial x}$ and $\frac{\partial^2 z}{\partial y \partial x}$ if $z = x^4y - 3x^2y^3$

#8. (13 pts.) Find the velocity, acceleration, and speed of a particle with position function
$$\vec{r}(t) = \left(2\cos(t), 3t, 2\sin(t)\right)$$

#BONUS. (10 pts.) Prove that
$$\lim_{(x, y) \to (0, 0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} = 2$$
Hint: You do NOT need $\epsilon$ and $\delta$ to do this problem – just a little algebra.

#1. (13 pts.) [A] State the linear approximation formula.
1. Find the linear approximation of the function \( f(x, y) = \frac{x}{y} \) at \( P(6, 3) \).

2. Use it to approximate \( f(5.95, 3.01) \). Give your answer to 9 decimal places.

3. Suppose that \( f \) is a differentiable function of \( x \) and \( y \), and \( g(r, s) = f(2r - s, s^2 - 4r) \). Use this table of values to calculate \( g_r(1, 2) \).

<table>
<thead>
<tr>
<th>( (0, 0) )</th>
<th>( f )</th>
<th>( g )</th>
<th>( f_x )</th>
<th>( f_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (0, 0) )</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>( (1, 2) )</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

4. (a) Find the maximum rate of change of \( f(x, y) = \frac{y^2}{x} \) at \( P(3, 5) \) and (b) the direction in which it occurs. Give exact answers to both parts.

5. In your textbook, Equation 6 is:

\[
\frac{dy}{dx} = \frac{F_x}{F_y}.
\]

Use this equation to find \( \frac{dy}{dx} \) if \( \sqrt{xy} = 1 + x^2 y \)

6. (a) At what point \( P \) does the function \( f(x, y) = 6x^2 + 3y^2 \) have a local minimum value?

(b) What is the local minimum value of this function at the point \( P \)?

7. Suppose \( R = [2, 5] \times [2, 6] \). Use a Riemann sum with \( m = 3 \) and \( n = 2 \) to estimate the value of \( \int_R \sqrt{64 - x^2 - y^2} \, dA \). Use the lower left corners of the sub-rectangles as your sample points. Give an exact answer.

8. Find the coordinates of the point \( Q \) on the plane \( 
\Pi : x - y + z = 4
\) that is closest to the point \( P(1, 2, 3) \). Exact answer.

#Bonus. Find the equation of the tangent plane, \( \Pi_{tgn} \), in Standard Form, to the surface \( z = 3x^2 - y^2 + 2x \) at \( P(1, -2, 1) \) using the technique described in Section 15.6.

NET #01: Using the techniques of Ch 16, find the volume of the solid beneath the surface

\[
\sum: z = 2x + y^2 + 4
\]

and over the region \( D: \) triangle with vertices \( (0, 0), (3, 3), \) and \( (6, 0) \). The units are centimeters.

NET #02: Neatly draw the region of integration, and then re-write the integral, changing the order of integration.

\[
\int_{x=2}^{x=6} \int_{y=x^2+1}^{y=5} f(x, y) \, dy \, dx
\]
NET #03: The vertices of a trapezoid are at \(O(0,0)\), \(A(2,3)\), \(B(4,3)\), and \(C(6,0)\). Additionally, it is known that the density function is \(\rho(x,y) = k\) \(k\) is a constant. Use these data to find the c.m. (center of mass). Note: Your final answer will “read:” “The center of mass is located at \((_,_,_\).” Also, plot your c.m. on your drawing. Use the techniques of Sect. 16.5 to do this problem.

NET #04: Find the surface area of that portion of the surface \(\Sigma: z = 36 - x^2 - y^2\) which is on or above the xy-plane. The units are centimeters. Report both the exact answer and an approximate answer rounded to 5 decimal places.

NET #05: Find the volume of the solid pyramid with vertices at \(O(0,0,0)\), \(A(1,0,0)\), \(B(1,2,0)\), \(C(0,2,0)\), \(D(0,0,1)\). [[Really, really big HINT: This is the pyramid from Problem #25, Sect. 16.7, p. 1067.]]

NET #06: The formula for a triple integral in cylindrical coordinates is

\[
\int_E f(x,y,z)\,dV = \int_{\theta=a}^{\theta=b} \int_{r=a}^{r=b} \int_{z=a}^{z=b} f(r\cos\theta,r\sin\theta,z) r\,dz\,dr\,d\theta ,
\]

Use this formula to evaluate \(\iiint_E x\,dV\) where \(E\) is the solid enclosed by the planes \(z = 0\) and \(z = 4\) and by the cylinders \(x^2 + y^2 = 1\) and \(x^2 + y^2 = 4\).

NET #07: Evaluate THE CURVILINEAR INTEGRAL \(\int_C \frac{y}{x}\,ds\) WHERE \(C\) IS THE CURVE GIVEN BY \(C: x = t, y = t^2, 0 \leq t \leq 10\).

NET #08: Evaluate THE CURVILINEAR INTEGRAL \(\int_C 2y^3\,dx + x^2\,dy\) WHERE \(C\) IS THE LINE SEGMENT FROM (2,4) TO (8,-3).

NET #09: A thin wire is bent into the shape of a semicircle with \(x = 4\cos(\pi t), y = 4\sin(\pi t), 0 \leq t \leq 1\). Suppose that the uniform linear density of the wire is \(\rho(x,y) = 3\) g/cm. Find the coordinates of the c.m. of the wire. Give both an exact answer and an approximation rounded to 1 decimal place. Graph the wire and the c.m.

NET #10: Evaluate \(\int_C \vec{F} \cdot d\vec{r}\) if \(\vec{F}(x,y,z) = \langle z, y, -x \rangle\) and \(\vec{r}(t) = \langle t, \sin(t), \cos(t) \rangle, 0 \leq t \leq \pi\).

NET #11: Suppose that \(\vec{F}(x,y) = y\vec{i} + (x+2y)\vec{j}\). Then I know that \(\vec{F}\) is conservative, since
\[
\frac{\partial}{\partial y}(y) = 1 = \frac{\partial}{\partial x}(x+2y).
\]

[A] Find the potential function \(f(x,y)\) such that \(\nabla f(x,y) = \vec{F}(x,y)\).

[B] Use the FTLI (Fundamental Theorem of Line Integrals) to evaluate \(\int_C \vec{F} \cdot d\vec{r}\) where \(C\) is the upper semicircle that starts at \((0,1)\) and ends at \((2,1)\).