I. Chain Rule - Case 2

\[ Z = f(x, y) \land x = g(s, t) \land y = h(s, t) \]
and everything is differentiable, then
\[
\frac{\partial Z}{\partial s} = \frac{\partial Z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial Z}{\partial y} \cdot \frac{\partial y}{\partial s}
\]
\[
\frac{\partial Z}{\partial t} = \frac{\partial Z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial Z}{\partial y} \cdot \frac{\partial y}{\partial t}
\]

B. Weird Example

\[ \frac{\partial W}{\partial t} = \frac{\partial W}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial W}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial W}{\partial z} \cdot \frac{\partial z}{\partial t} \]
\[
\frac{\partial W}{\partial s} = \frac{\partial W}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial W}{\partial z} \cdot \frac{\partial z}{\partial s}
\]
\[
\frac{\partial W}{\partial u} = \frac{\partial W}{\partial y} \cdot \frac{\partial y}{\partial u} + \frac{\partial W}{\partial z} \cdot \frac{\partial z}{\partial u}
\]

C. Examples of "trees" pp. 970-1

II. Implicit Differentiation (p. 972)

A. Start with an equation involving \( x, y \). "Move" everything over to one side & call it \( F(x, y) \); thus, our original eqn. now looks like this
\[ F(x, y) = 0 \]

B. Example
\[ x^2 = 2y - y^2 + 14 \quad \rightarrow \quad x^2 + y^2 - 2y - 14 = 0 \]
and identify \( x^2 + y^2 - 2y - 14 \) as \( F(x, y) \).

So what I've got is

\[
F(x, y) = 0
\]

\[
x^2 + y^2 - 2y - 14 = 0
\]

i.e., \( F(x, y) = x^2 + y^2 - 2y - 14 \)

\[\underline{My \ original \ problem \ equation \ F(x, y) = 0} \]

\[\square \text{One of the benefits of this approach is that if } F \text{ is differentiable, then} \]

\[
\frac{dF}{dx} = \frac{\partial F}{\partial x} \frac{dx}{dx} + \frac{\partial F}{\partial y} \frac{dy}{dx}
\]

\[\square \text{Chain Rule Case 1} \]

And on the other hand

\[
\frac{d0}{dx} = 0
\]

So \( F(x, y) = 0 \)

\[
\frac{dF}{dx} = \frac{d0}{dx}
\]

\[
\frac{\partial F}{\partial x} \frac{dx}{dx} + \frac{\partial F}{\partial y} \frac{dy}{dx} = 0
\]

\[
\frac{dF}{dx} + \frac{dF}{dy} \cdot \frac{dy}{dx} = 0
\]

And we have a nifty formula for \( \frac{dy}{dx} \):