I. Composition of Functions — [Review]

A. Example: If \( f(x) = 3x - 5 \) and \( g(x) = 2x + 8 \), then \( f \circ g(x) \) is the composite function, and it is defined as \( f \circ g(x) = f(g(x)) = f(2x + 8) = 3[2x + 8] - 5 = 6x + 19 \). Therefore, \( f \circ g(x) = 6x + 19 \).

B. In general, \( f \circ g(x) \neq g \circ f(x) \).

C. Let's look at our example "A."

\[ g \circ f(x) = g(f(x)) = g(3x - 5) = 2[3x - 5] + 8 = 6x - 2 \]

Thus \( f \circ g(x) \neq g \circ f(x) \) for this example.

II. Consider the COMPOSITION of TWO DIFFERENTIAL OPERATORS:

A. Example: If \( (D+4)(y) \) and \( (D-1)(y) \)

\[ (D+4)(y) = \frac{dy}{dx} + 4y \]

\[ (D-1)(y) = \frac{dy}{dx} - y \]

B. Diff. Eqs.: \( (D+4)(y) = 0 \) \( \iff \) \( \frac{dy}{dx} + 4y = 0 \)

which has the solution \( y = c_1 e^{-4x} \).

Also \( (D-1)(y) = 0 \) \( \implies \) \( y = c_2 e^x \) is the Gen. Sol.

C. Consider \( y'' + 3y' - 4y = 0 \). Solve it.
Soll
\[ m^2 + 3m - 4 = 0 \] Aux Ex.
\[ (m+4)(m-1) = 0 \] \{ -4, 1 \}
\[ 3 \text{ GenSol. } y = c_1 e^{-4x} + c_2 e^x \]

Further Consider: \( y'' + 3y' - 4y = 0 \).

Trans into \( L(y) = 0 \):
\[ \left[ D^2 + 3D - 4 \right] (y) = 0 \]
So
\[ \left[ D^2 + 3D - 4 \right] (y) = 0 \]
\[ \left[ (D+4)(D-1) \right] (y) = 0 \].

Now look at \( (D-1)(y) = 0 \): has solution \( y = c_1 e^x \)
Now look at \( (D+4)(c_1 e^x) = \frac{d}{dx} \left( c_1 e^x \right) + 4 \left( c_1 e^x \right) \)
\[ = c_1 e^x + 4c_1 e^x = 5c_1 e^x = c e^x \]
And similarly \( (D+4)(y) = 0 \) \( y = c_2 e^{-4x} \)
and \( (D-1) \left( c_2 e^{-4x} \right) = \frac{d}{dx} \left( c_2 e^{-4x} \right) - \left( c_2 e^{-4x} \right) \)
\[ = -4c_2 e^{-4x} - c_2 e^{-4x} = c e^{-4x} \]
\[ \therefore \left[ (D-1)(D+4) \right] (y) = 0 \] \( y = c e^{-4x} \)
\[ \left[ (D+4)(D-1) \right] (y) = 0 \] \( y = c e^x \)

However, If I look at \( \left[ D^2 + 3D - 4 \right] (y) = 0 \)
and factor the DiffOp. as \( (D+4)(D+3) \)
Then I can get the GenSol.
to \([D^2+3D-4](y) = 0\) by forming the lin.combo. of the sols to the factors \((D+4)(y)=0\) and \((D-1)(y)=0\)

III \hspace{1cm} \text{Page 158: §4.5. "ANNIHILATOR"}

A \hspace{1cm} \text{If } L(y) \text{ is a diff.eq., and if } y=f(x) \text{ is a function, and if}

\[L(f')(x) = 0, \ldots\] \hspace{1cm} \text{THEN we say that}

"L \text{ annihilates } f." \hspace{1cm} \text{or "L is an Annihilator of } f." \hspace{1cm}

B \hspace{1cm} \text{Finding Annihilators.}

1 \hspace{1cm} f(x) = x^3 \hspace{1cm} \text{and } L = D

\[D(x^3) = 3x^2\]
\[D^2(x^3) = 6x\]
\[D^3(x^3) = 6\]
\[D^4(x^3) = 0\]
\[D^5(x^3) = 0\]

So obviously \(D^{(n)} \text{ annihilates } ax^n\)

So \(D^3(2x^2+5x-6) = 0\) \(\text{p159}\)
Recall if \( y'' + 4 = 0 \) (*)

\[
\begin{align*}
\text{Soln:} & \\
\text{1.} & \quad m^2 + 4 = 0 \\
\text{2.} & \quad m^2 = -4 \\
& \quad m = \pm \sqrt{-4} = \pm 2i \quad \{2i, -2i\}
\end{align*}
\]

**Type III:**

3. \( y = c_1 \cos(2x) + c_2 \sin(2x) \)

If, for example \( c_1 = 1 \) and \( c_2 = 0 \)

\( y = \cos(2x) \) is a sol to (*)

\[
\begin{align*}
\therefore \quad (D^2 + 4)(\cos(2x)) &= 0 \\
\text{also} \quad (D^2 + 4)(\sin(2x)) &= 0
\end{align*}
\]

\[
\therefore \quad D^2 + 4 \text{ ANNIHILATES both } \cos(2x) \text{ and } \sin(2x).
\]

So we can Annihilate polynomial functions and we can Annihilate sine and cosine functions...

How do we Annihilate exponential functions? ...