Inverses — "Opposite Operations in Opposite Order."

A  \[ G = 3 + \frac{a}{4} \]
\[ \frac{-3}{-3} \]
\[ 3 = \frac{a}{4} \]
\[ 4 \cdot 3 = 4 \cdot \frac{a}{4} \]
\[ 12 = a \]
\[ \{12\} \]

B  \[ f(x) = 3x + 5 \]
\[ f^{-1}(x) = \frac{x-5}{3} \]

2 "Classic" Way
\[ f(x) = 3x + 5 \]
\[ y = 3x + 5 \]
\[ x = 3y + 5 \] ("SWAP")
\[ x - 5 = 3y \] (Solve for \(y^\prime\))
\[ \frac{x - 5}{3} = y \]
\[ y = \frac{x - 5}{3} \]
\[ f^{-1}(x) = \frac{x - 5}{3} \] (Put the inverse "tag" onto the "rule,")
II. Inverse Sine Function

\[ y = \sin^{-1}(x) \]
\[ y = \arcsin(x) \]

![Graph of \( y = \sin^{-1}(x) \) restricted to \([-\pi/2, \pi/2]\)]

III. Composition of functions

[A] Example: If \( f(x) = 2x + 3 \) and \( g(x) = x^2 - 1 \), then

\[ f \circ g(x) = f(g(x)) = 2(g(x)) + 3 \]
\[ = 2(x^2 - 1) + 3 = 2x^2 - 2 + 3 \]
\[ = 2x^2 + 1. \]

\[ \therefore f \circ g(x) = 2x^2 + 1 \]
**Example:** If \( f(x) = 3x + 5 \) and \( g(x) = \frac{x - 5}{3} \)

then \( f \circ g(x) = 3(g(x)) + 5 = 3 \left( \frac{x - 5}{3} \right) + 5 = x \)

\( \therefore f \circ g(x) = x \)

**Define:** \( 1_x(x) = x \) (The "identity function")

**In the case above** \( f \circ g(x) = 1_x(x) \)

Think — in algebra, \( a \cdot a^{-1} = a \cdot \frac{1}{a} = 1 \)

This is the "justification" for the notation

\( f \circ f^{-1} = 1_x \)

\( f(x) = 3x + 5 \quad f^{-1}(x) = \frac{x - 5}{3} \)

**IV** Composition of Trig Functions w/ Inverses

**A** \( \sin(\sin^{-1}(\frac{1}{2})) = \sin(\frac{\pi}{6}) = \frac{1}{2} \) Nice.

**B** \( \sin^{-1}(\sin(\frac{3\pi}{8})) = \sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{3} \) Weeps!

Why is this.

(Class ends here!)

PS Look on CourseCompass for BOTH quizzes & homework.
1. \[ \sin^{-1}(\sin \left(\frac{2\pi}{3}\right)) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \]

2. If \( \theta \) is in the interval \( \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \) then \( \sin^{-1}(\sin(\theta)) = \theta \).

3. \[ \sin^{-1}(\sin \left(\frac{\pi}{6}\right)) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \]

4. \[ \sin^{-1}(\sin \left(\frac{5\pi}{6}\right)) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \]

5. \[ \sin^{-1}(\sin \left(\frac{2\pi}{9}\right)) = \frac{2\pi}{9} \] Why? \( \frac{\pi}{2} = \frac{4.5\pi}{9} \) \( \frac{2\pi}{9} \) \( \frac{5\pi}{9} \) \( \frac{5\pi}{9} \) \( \frac{2\pi}{9} \) \( 180^\circ = 40^\circ \) \( \frac{x}{2} \) 

6. \[ \sin^{-1}(\sin \left(\frac{5\pi}{9}\right)) = \frac{4\pi}{9} \]

Regardless, \( \sin(\sin^{-1}(\theta)) = \theta \) Always.