§ 7.3: Review for Test #1.

A. From the MML:
\[
\frac{\cot(v) - 1}{\cot(v) + 1} = \frac{1 - \tan(v)}{1 + \tan(v)} \quad \text{Establish ID.}
\]

Est.

1. Work:
\[
\frac{\cot(v) - 1}{\cot(v) + 1} = \frac{\frac{1}{\tan(v)} - 1}{\frac{1}{\tan(v)} + 1} = \frac{1 - \tan(v)}{\tan(v)} \quad \frac{1}{1 + \tan(v)}
\]

= \frac{1 - \tan(v)}{1 + \tan(v)}

2. \[\cot(v) - 1 = \frac{1 - \tan(v)}{1 + \tan(v)}\]

B. #18 p. 451 Factor & Simplify:
\[
\frac{\cos^2(\theta)-1}{\cos^2(\theta)-\cos(\theta)}
\]

Soll
\[
\frac{\cos^2(\theta)-1}{\cos^2(\theta)-\cos(\theta)} = \frac{(\cos(\theta)+1)(\cos(\theta)-1)}{\cos(\theta)(\cos(\theta)-1)}
\]

= \frac{\cos(\theta)+1}{\cos(\theta)}

C. #78 (p. 452) Est. ID.
\[
\frac{\sec^2(v)-\tan^2(v)+\tan(v)}{\sec(v)}
\]

= \sin(v) + \cos(v)
$$\sec^2(v) - \tan^2(v) + \tan(v) = \frac{1}{\sec(v)} - \frac{\sin^2(v)}{\cos^2(v)} + \frac{\sin(v)}{\cos(v)}$$

$$= \frac{1 - \sin^2(v)}{\cos^2(v)} + \frac{\sin(v)}{\cos(v)}$$

$$= \frac{1}{\cos(v)}$$

$$\frac{\cos(v) + \sin(v)}{\cos(v)} = \cos(v) + \sin(v) = \sin(v) + \cos(v)$$

II §7.2. Example 6 "type" p. 443.

A #59 (p. 445). Write as algebraic expression in "u." \(\tan(\sin^{-1}(u))\)

Sol I \(\Theta = \sin^{-1}(u)\) \(\Rightarrow \sin(\Theta) = u\)

2 \(\sin^2(\Theta) + \cos^2(\Theta) = 1 \Rightarrow u^2 + \cos^2(\Theta) = 1\)

\(\therefore \cos^2(\Theta) = 1 - u^2\) \(\cos(\Theta) = \pm \sqrt{1-u^2}\) use "±"

3 \(\therefore \tan(\sin^{-1}(u)) = \tan(\Theta) = \frac{\sin(\Theta)}{\cos(\Theta)} = \frac{u}{\sqrt{1-u^2}}\)

4 \(\tan(\sin^{-1}(u)) = \frac{u}{\sqrt{1-u^2}}\)
§ 6.2: p 371: part of # 123. (You must read the entire text of the problem in the book.)

\[ R = 8 \text{ mi/hr} \]

\[ T(\theta) = 1 + \frac{2}{3 \sin(\theta)} - \frac{1}{4 \tan(\theta)} \quad 0^\circ < \theta < 90^\circ \]

(a) \( T(30^\circ) \) How long is Sally on paved Rd.

\[ T(30^\circ) = 1 + \frac{2}{3 \left( \frac{1}{2} \right)} - \frac{1}{4 \left( \frac{1}{\sqrt{3}} \right)} \]

\[ = 1 + \frac{2 \sqrt{3}}{2} - \frac{1}{4 \sqrt{3}} = 1 + 2 \cdot \frac{2}{3} - 1 \cdot \frac{\sqrt{3}}{4} = 1 + \frac{4}{3} - \frac{\sqrt{3}}{4} \]

\[ = \frac{12 + 16 - 3 \sqrt{3}}{12} = \frac{28 - 3 \sqrt{3}}{12} \approx 1.900 \, 320 \, 631 \, \text{hrs} \]

\[ D_T = D_s + D_R = R_s T_s + R_r T_r \]

\[ D_T = 8 - 2x + 2 \sqrt{x^2 + 1} \]

If \( \theta = 30^\circ \)

\[ \tan(30^\circ) = \frac{1}{\sqrt{3}} \]

\[ \tan(30^\circ) = \frac{1}{x} \]

\[ x = \sqrt{3} \]
\[ x = \sqrt{3} \quad D_T = 8 - 2x + 2\sqrt{x^2 + 1} \]

\[ D_T = 8 - 2\sqrt{3} + 2\cdot 2 = (12 - 2\sqrt{3}) \]

\[ D_T = R_s T_s + R_e T_e \]

Let \( t = \) time on road

\[ 12 - 2\sqrt{3} = \frac{3}{4} (28 - 3\sqrt{3} - t) + 8t \]

\[ 12 - 2\sqrt{3} = \frac{28 - 3\sqrt{3}}{4} - 3t + 8t \]

\[ 5t = 12 - 2\sqrt{3} - 7 + \frac{3}{4} \sqrt{3} \]

\[ 5t = 5 - \frac{5\sqrt{3}}{4} \]

\[ t = 1 - \frac{\sqrt{3}}{4} \text{ hrs. } \approx 0.5669872981 \text{ hrs} \]

\[ t \approx 0.6 \text{ hrs } \approx 36 \text{ min.} \]

**Note:** This problem is a Word Problem, so a WPA is required.

**5.** The time she spent on the road was approx. 36 min.

IV. §6.3: p. 385: #119. Trip time

\[ T(\theta) = 5 - \frac{5}{3 \tan(\theta)} + \frac{5}{\sin(\theta)} \quad 0 < \theta < \frac{\pi}{2} \]

The time she spent on the road was approx. 36 min.

\[ \tan(\theta) = \frac{500}{1500} = \frac{1}{3} \]

\[ S = \sqrt{500^2 + 1500^2} = \sqrt{100^2 \cdot 5^2 + 100^2 \cdot 15^2} = 100\sqrt{25 + 225} \]

\[ R_p = 300 \text{ ft/min.} \quad R_e = 100 \text{ ft/min.} \]
\[ = 100\sqrt{250} = 500\sqrt{10} = 500\sqrt{10} \text{ ft} \]

Length of "Snail path" is \(500\sqrt{10}\) ft.

\[ D = RT \implies T = \frac{D}{R} = \frac{500\sqrt{10} \text{ ft}}{100 \text{ ft/min}} = 5\sqrt{10} \text{ min} \]

:. The time is \(5\sqrt{10}\) min or approx 15.8 min.

\[ 5\sqrt{10} \approx 15.8113883 \] Calculator result.

\[ \hat{\theta} = \cos^{-1}(x) \]

\[ 0 \leq \cos^{-1}(x) \leq \pi \]

\[ \hat{\theta} \] So if \( \hat{\theta} \) STARTS out between \( 0 \) and \( \pi \), that's where it ends up.

\[ \theta \]

Unit circle w/ range of arccosine fun. emphasized

\[ \text{Graph of } y = \cos^{-1}(x) \]

\[ \cos^{-1}(\cos(\frac{4\pi}{5})) \]

A

\#37 \( (p.438) \)

\[ \cos^{-1}(\cos(\frac{4\pi}{5})) \]

Sol. 1

\[ \cos(\frac{4\pi}{5}) \]

2

\[ \cos^{-1}(\cos(\frac{4\pi}{5})) = \frac{4\pi}{5} \]

B

Follow-up - Related problems.

1

\[ \cos^{-1}(\cos(\frac{6\pi}{5})) = \frac{4\pi}{5} \]

2

\[ \cos^{-1}(\cos(\frac{4\pi}{3})) = \frac{2\pi}{3} \]

When \( \hat{\theta} \) starts out outside \([0,\pi]\), then it must "end up" within \([0,\pi]\).

If \( \hat{\theta} \) starts out \text{OUTSIDE} \([0,\pi]\), then it must "end up" within \([0,\pi]\).