4.5 (Context)

218 - Descartes's Rule of Signs - not required, but useful.

The "generic" polynomial function

\[ P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0 \]

where \( n \in \mathbb{Z}^+ \) (pos. integer)

\[ a_n \neq 0 \]

If additionally (he says) \( a_0 \neq 0 \) then "possible rational roots" of

\[ a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 = 0 \]

are of the form \( \frac{p}{q} \) where \( p \) is a factor of \( a_0 \)

and (2) is a factor of \( a_n \).

P. 223 - Bounds on Zeros

\[ A = \text{Max} \{ 1, |a_0| + |a_1| + |a_2| + \ldots + |a_{n-1}| \} \]

\[ B = 1 + \text{Max} \{ |a_0|, |a_1|, |a_2|, \ldots, |a_{n-1}| \} \]

\[ M = \text{min} \{ A, B \} \]

Thm: \(-M \leq \text{any zero of } P(x) \leq M\)
\[ f(x) = 3x^3 - 2x^2 + x + 2 \]

Find "window" to "catch" all the zeros.

**Soln:**
\[
A = \max \left\{ \frac{1}{3}, |a_0| + \cdots + |a_{n-1}| \right\} \\
= \max \left\{ 1, 2 + 1 + 2 \right\} = \max \left\{ 1, 5 \right\} = 5
\]

\[
B = 1 + \max \left\{ |a_0|, |a_1|, \ldots, |a_{n-1}| \right\} \\
= 1 + \max \left\{ \frac{1}{3}, 2, 1, 2 \right\} = 1 + 2 = 3
\]

\[
M = \min \left\{ A, B \right\} = \min \left\{ 5, 3 \right\} = 3
\]

\[ -3 \leq \text{zero} \leq 3 \]

This is the "x-window" set your calculators accordingly to "catch" all the zeros.

**E.** (p. 224) Intermediate Value Theorem

Suppose \( f(a) \) and \( f(b) \) have different signs.
Then there is some number \( c, \ a < c < b \), such that \( f(c) = 0 \).

**Ex. 9, p. 224.** Show that \( f(x) = x^5 - x^3 - 1 \) has a zero between 1 and 2.

**Soln.** \( f(1) = -1 \) and \( f(2) = 32 - 8 - 1 > 0 \)

\[ \therefore \text{by the IVT, there exists a number } c, \ 1 < c < 2 \text{ such that } f(c) = 0. \]
§ 4.5: p. 226: #12 \quad f(x) = -4x^3 + 5x^2 + 8,

x+3.

(a) Use Remainder Thm. to find remainder when
f(x) is divided by x - c

(b) Use Factor theorem to determine whether x - c is
a factor of f(x)

Solu. \quad a \quad 1

\[ \begin{array}{c|cccc}
& -4 & 5 & 0 & 8 \\
\hline 
-3 & & & & \\
\hline 
& 12 & -51 & 153 \\
& -4 & 17 & -51 & 161 \\
\end{array} \]

2. The remainder is 161
\[ \text{i.e. } f(-3) = 161 \]

x + 3 is not a factor of \( f(x) = -4x^3 + 5x^2 + 8 \).

§ 4.6 Complex Numbers // Zeros.

\[ i^2 = -1 \]

\( i = \sqrt{-1} \)

Def. 5

to be continued.