I returned IC-NET#07 and passed out NET#08 (due Mon).


The characteristic, or defining, property of an arithmetic sequence (AS) is that to "move" from one term to the next, you ADD a fixed amount (number).

Example: Let $a_1 = 5$ and $d = 3$.

Thus the seq. looks like this:

$$5, 8, 11, 14, 17, 20, 23, \ldots$$

So

$$a_1 = 5$$
$$a_2 = 5 + 3$$
$$a_3 = 5 + 3 + 3$$
$$a_4 = 5 + 3 + 3 + 3$$
$$a_5 = 5 + 3 + 3 + 3 + 3$$
$$a_{10} = 5 + 9(3)$$

They call this "d" because it is the common difference.

Formulas for the "n-th" term of an (AS)

$$a_n = a_1 + (n-1)d$$

$p.793$
B. Find the sum of the "first n-terms" of an AS.

\[ \sum_{k=1}^{n} (a_1 + (k-1)d) = S_n \]

Here's the symbolism for the sum—using the Sigma Notation.

**WORK**

1. Write \( S_n \) down twice—forward and backwards—Add them all up—
2. Recognize you are adding the same thing over & over—
3. Divide by 2:

\[
\begin{align*}
S_n &= a_1 + (a_1+d) + (a_1+2d) + \ldots + (a_1+(n-2)d) + (a_1+(n-1)d) \\
S_n &= (a_1+(n-1)d) + (a_1+(n-2)d) + (a_1+(n-3)d) + \ldots + (a_1+d) + a_1 \\
2S_n &= (2a_1 + (n-1)d) + (2a_1+(n-1)d) + (2a_1+(n-2)d) + \ldots + (2a_1+(n-3)d) + (2a_1+(n-2)d) \\
2S_n &= n \cdot (2a_1 + (n-1)d)
\end{align*}
\]

\[ S_n = \frac{n}{2} (2a_1 + (n-1)d) \]

Sum of 1st "n-terms" of an AS.

C. "Spin-Off" Formula.

\[ S_n = \frac{n}{2} (2a_1 + (n-1)d) = \frac{n}{2} (a_1 + a_1 + (n-1)d) \]

\[ = \frac{n}{2} (a_1 + (a_1 + (n-1)d)) = \frac{n}{2} (a_1 + a_n) \]

\[ S_n = n \left( \frac{a_1 + a_n}{2} \right) \]
D Put Theory to Practice!

1. §12.2: p. 796: #8: Show that. Find \( d \). List first 4 terms.
   \[ \{a_n\} = \{4 - 2n\} \]

   **SOLN:**
   
   i. 1st 4 terms: \[2, 0, -2, -4\]
   
   ii. \( d = -2 \)
   
   iii. \( \{a_n\} \) is AS because \( a_1 = 2 \) and \( d = -2 \)

2. §12.2: p. 796: #22: Find the 80th term of the AS
   \[-1, 1, 3, \ldots\]

   **SOLN:**
   
   1. \( a_1 = -1 \) \( d = 2 \)
   
   2. \[ a_{80} = a_1 + 79d \]
   
   \[ = (-1) + 79(2) \]
   
   \[ = 157 \]

3. §12.2: p. 796: #41: Find the sum: \( 5 + 9 + 13 + \ldots + 49 \)
   
   **SOLN:**
   
   i. \( a_1 = 5, \ n = ? \), \( d = 4 \)
   
   ii. \[ 49 = a_n = a_1 + (n-1)d = 5 + (n-1)4 \]
5 + (n-1)4 = 49  \quad \begin{array}{c}
5 + 4(n - 1) = 49 \\
4n - 4 + 5 = 49 \\
4n = 44 \\
\frac{4n}{4} = \frac{44}{4} \\
n - 1 = 11 \\
\boxed{n = 12}
\end{array}

Also, \( a_1 = 5 \), \( n = 12 \), \( d = 2 \)

\begin{align*}
S_n &= n \left( \frac{a_1 + a_n}{2} \right) \\
S_{12} &= 12 \left( \frac{5 + 49}{2} \right) = 12 \times 27 \\
S_{12} &= 324
\end{align*}

\[\text{or}\]

\[S_n = \frac{n}{2} \left[ 2a_1 + (n-1)d \right]\]

\begin{align*}
S_{12} &= 6 \left[ 10 + 11 \times 2 \right] = 6 \times 54 = \boxed{324}
\end{align*}

\[\text{or}\]

\[S_{40} = 20 \left[ 30 + 39 \times 2 \right] = 20 \times 108 = \boxed{2160}\]
There are 2,160 seats in the section.

§12.3: Geometric Seq. & Series. p. 797.

A. The characteristic property of a GS is that to move from one term the next, you multiply by a fixed number.

Example: $a_1 = 5, r = 2$

$5, 10, 20, 40, 80, \ldots$

$\frac{a_{n+1}}{a_n} = r$ in Symbols!

B. Formula for $n$-th term of a GS.

$a_1 = a_1$
$a_2 = a_1r$
$a_3 = a_1rr$
$a_4 = a_1rrr$

Generalize

$\begin{cases} a_n = a_1r^{n-1} \text{ (GS)} \\ a_n = a_1 + d(n-1) \text{ (Arithmetic)} \end{cases}$

Compare!