Level Curves. §14.1 (p. 952)

A Maple Worksheet.

B

5 ft contours.

Level Surfaces, p.953: §14.1

A 1 \( x^2 + y^2 = r^2 \) in 2-Dim. \( r = \text{radius} \).

2 \( x^2 + y^2 + z^2 = r^2 \) in 3-Dim. \( r = \text{radius} \).

3 \( x^2 + y^2 + z^2 + w^2 = r^2 \) in 4-Dim. \( r = \text{radius} \).

Take the level surface for 3 when \( W = 0 \), I get a sphere centered at the origin, of radius \( r \).

Classifications of sets in "Space."

A In 1-dim an interval might be open, closed, or partially open/closed.

Interval \((a,b)\), \([a,b]\), \((a,b]\), \([a,b)\)

We can have an open interval of radius \(\varepsilon > 0\) centered at \(x_0\) --

Let's call the interval \(I\) \( \rightarrow I = (x_0 - \varepsilon, x_0 + \varepsilon) \)

1 In interval notation, how do we write \(I\) ?

\( I = \{ x \mid x > x_0 - \varepsilon \text{ and } x < x_0 + \varepsilon \} \)

\( x_0 - \varepsilon = \varepsilon \) \( \therefore \ varepsilon = x_0 - \varepsilon \)
Play w/ the conditions on set I.

\[
\begin{align*}
  x > x_0 - \varepsilon \\
  \varepsilon > x_0 - x \\
  x - x_0 < \varepsilon \\
  x_0 - x < \varepsilon \\
  \text{and} \\
  x - x_0 < \varepsilon
\end{align*}
\]

\[\therefore |x_0 - x| < \varepsilon\]

\[\therefore I = \{ x \mid |x_0 - x| < \varepsilon \} \quad \text{or} \quad \{ x \mid |x - x_0| < \varepsilon \} \]

Let's drop all this "set mumbo-jumbo" and write

\[|x - x_0| < \varepsilon.
\]

Recall

\[|A| = \begin{cases} 
  -A & \text{if } A < 0 \\
  A & \text{if } A > 0
\end{cases}
\]

\[\therefore \sqrt{(x - x_0)^2} < \varepsilon. \text{ This is } (x_0 - \varepsilon, x_0 + \varepsilon).
\]

The boundary or end points of this interval could be written as

\[\pm \sqrt{(x - x_0)^2} = \varepsilon.
\]

or \[(x - x_0)^2 = \varepsilon^2.
\]

In 2-dim: \((x - x_0)^2 + (y - y_0)^2 = \varepsilon^2 \leftarrow \text{circle}
\]

Open Disk (inside) \((x - x_0)^2 + (y - y_0)^2 < \varepsilon^2
\)

\[\text{open disk centered at } (x_0, y_0) \text{ with radius } \varepsilon. \quad \text{(boundary NOT included)}\]