§ 14.4 Chain Rule.

A

1. \( w = f(x, y, z) \quad x = x(r, s), \; y = y(r, s) \)
   \( \text{and} \quad z = z(r, s) \)

a. Final variable (Outcome of the "process") — \( w \)

   Intermediate Variables — \( x, y, z \)

   Initial variables — \( r, s \).

b. [Diagram showing relationships between variables]

   "TREE"

   usual method.

c. Taking a path DOWN — is a derivative!

   One path down followed by another path down — is the product of derivatives.

\[ \frac{\Delta w}{\Delta x} \quad \frac{\Delta w}{\Delta z} \]

\[ \frac{\Delta x}{\Delta r} \quad \frac{\Delta y}{\Delta r} \quad \frac{\Delta z}{\Delta s} \]

A total path, such as that from \( w - x - r \) we'll call a "thread."

Now to find, for instance \( \frac{\Delta w}{\Delta s} \), we ADD threads which connect \( w \) to \( s \).
\[ \frac{dw}{ds} = \frac{dw}{dx} \frac{dx}{ds} + \frac{dw}{dy} \frac{dy}{ds} + \frac{dw}{dz} \frac{dz}{ds} \]

2. For example, \( W = f(x, y, z) \)
\( x = r + 2s, \quad y = 2s - r, \quad z = s - r. \)

and \( f(x, y, z) = x + y^2 - z^2 \)

\[ W = \]

Find \( \frac{dw}{dr} \)

**Solution**

\[ \frac{dw}{dr} = \frac{dw}{dx} \frac{dx}{dr} + \frac{dw}{dy} \frac{dy}{dr} + \frac{dw}{dz} \frac{dz}{dr} \]

\[ = (1 - z) \cdot 1 + 2y (-1) + (-x) (-1) \]

\[ = 1 - z - 2y + x \]

\[ = \begin{vmatrix} 1 - z - 2y + x \end{vmatrix} = \]

or

\[ \begin{vmatrix} x - s + r - 4s + 2r + r + z \end{vmatrix} \]

\[ = \begin{vmatrix} -3s + 4r \end{vmatrix} \]

\[ = \begin{vmatrix} 1 + 4r - 3s \end{vmatrix} \]

13. We have something like \( x^2 + y^2 + z^2 = 4 \)

This is the LEVEL SURFACE \( x^2 + y^2 + z^2 - 4 = 0 \)

of a hypersphere

\[ w = F(x, y, z) \]

Where \( F(x, y, z) = x^2 + y^2 + z^2 - 4 \)

And the level surface is \( F(x, y, z) = 0 \)