We reviewed §14.7: p. 1018: #13, which will be posted on the "Helpful Hints" page.

Since the next test is Mon 3-29 ½ Tue 3-30 and I made no announcement in class, I'm extending the redemption time till Wed 3/24. (Added after class).

§§14.7: Abs. Max/Min. Value on a Closed (contains bdry)
and Bounded (contained in some circle w/center @ origin) Region. (P. 1015)

$f(x,y)$ is cont. on a Closed & Bounded Region $R \subset \mathbb{R}^2$

Then $f$ has abs. max & abs min. values on $R$.

Program:

1. List all interior pts of $R$ at which $f$ may have local max/min & evaluate.
   - Try to use 2nd der. test - modified - just 1st partials!

2. List all bdry pts at which $f$ may have loc. max/min. values & evaluate.
   - Boils down to one-or-more Calc I problems.

3. Scan these lists of values for the biggest (abs. max. val.) and the smallest (abs. min. val.).

Authors’ Example #5: p. 1015

Find abs. max/min values of $f(x,y) = 2 + 2x + 2y - x^2 - y^2$
on $R$: Triangular Reg. in QI bdd by $x = 0$, $y = 0$ & $y = 9-x$.


2. Look in $\text{int}(R)$ - Try 2nd Der. Test. (Mod)
   - Just set 1st partials equal to zero & solve $f_x = 2 - 2x$, $f_y = 2 - 2y$
   - Set $0 = f_x = 2 - 2x \Rightarrow (x = 1)$ & $0 = f_y = 2 - 2y$
   - $\Rightarrow (y = 1)$
   - $f(P_0) = 2 + 2 + 2 - 1 - 1 = 6 - 2$
\[ f(x, y) = 2 + 2y - y^2 \]

3. a. on \( x = 0 \):

\[ f^* = 2 + 2y - y^2 \]

Find abs max/min on \([0, 9]\)

\[ f^* = 2 - 2y \]

\[ f^* = 2 - 2y \text{ set } y = 0 \implies y = 1 \]

\[ f^*(1) = 2 + 2 - 1 = 3 \]

and \( f^*(0) = 2 \) and \( f^*(9) = 2 + 18 - 81 = -61 \)

at ep's,

\[ f(0, 0) = 2, \ f(0, 1) = 3, \ f(0, 9) = -61 \]

b. on \( y = 0 \):

\[ f^* = 2 + 2x - x^2 \]

Find abs max/min on \( 0 \leq x \leq 9 \).

\[ f^* = 2 - 2x \text{ set } x = 0 \implies x = 1 \]

\[ f^*(1) = 2 + 2 - 1 = 3 \]

Now @ ep's:

\[ f^*(0) = 2 \]

and \( f^*(9) = 2 + 18 - 81 = -61 \)

\[ f(1, 0) = 3, \ f(0, 0) = 2 \ (\text{already knew that}) \]

\[ f(9, 0) = -61 \]

C. on \( y = 9 - x \) \( 0 \leq x \leq 9 \)

\[ f^* = 2 + 2x + 2(9 - x) - x^2 - (9 - x)^2 = 2 + 2x + 18 - 2x - x^2 \]

\[ = -81 + 18x - x^2 \]

\[ f^* = -61 + 18x - 2x^2 \]

and \( f^* = 18 - 4x \)

Set \( 0 = f^* = 18 - 4x \implies x = \frac{18}{4} = \left( \frac{9}{2} \right) \)

and

\[ f^* \left( \frac{9}{2} \right) = -61 + 18 \left( \frac{9}{2} \right) - 2 \left( \frac{81}{4} \right) = -61 + 81 - \frac{81}{2} \]

\[ = -61 + \frac{81}{2} = -\frac{121}{2} + \frac{81}{2} = -\frac{41}{2} \]

At ep's:

\[ f^*(0) = -61 \]

and \( f^*(9) = -61 + 18(9) - 2(81) \)

\[ = -61 \]

\[ f^* \left( \frac{9}{2} \right) = -\frac{41}{2}, \ f(0, 9) = -61, \ f(9, 0) = -61 \]

CHART

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(f(x,y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1,1))</td>
<td>4</td>
</tr>
<tr>
<td>((0,0))</td>
<td>2</td>
</tr>
<tr>
<td>((0,9))</td>
<td>-61</td>
</tr>
<tr>
<td>((1,0))</td>
<td>3</td>
</tr>
<tr>
<td>((9,0))</td>
<td>-61</td>
</tr>
<tr>
<td>(\left( \frac{9}{2}, \frac{9}{2} \right))</td>
<td>-\frac{41}{2}</td>
</tr>
</tbody>
</table>

5. \(f\) has an abs max value of 4 at \((1,1)\) and an abs min value of -61 at \((0,9)\) and \((9,0)\) on the region \(R\).