

I. Reviewed additions made to yesterday's notes before posting.

II. § 15.4: Triple Integrals (p. 1082)

A. Read "Intro" pp. 1082-3.

B. Definition: Volume (p.1083) If D is a closed and bounded region (Domain) in 3-space, then

\[ V = \iiint_D dV \]

\[ \Delta V = \Delta x \Delta y \Delta z \]

Use Fubini.

C. We have a solid bdd by \( x = 1 \), \( y = 2 \), \( z = 3 \).

Find \( V \).

Find order of integration.

In how many ways can one arrange the symbols "\( dx \)," "\( dy \)," and "\( dz \)?

\[ 3 \times 2 \times 1 = 6 \]

Here is one order of integration

@ For \((x,y)\) fixed, \(z\) varies from \( z=0 \) to \( z=3 \)

@ For \(x\) fixed, \(y\) varies from \( y=0 \) to \( y=2 \)

@ \(y\) varies from \( x=0 \) to \( x=1 \)
\[ V = \int_{x=0}^{x=1} \int_{y=0}^{y=2} \int_{z=0}^{z=3} \, dz \, dy \, dx = \text{[finish]} \]

### III. Properties. (Read)

1. **Of Double Integrals** (p.1062) (§15.1).
2. **Of Triple Integrals** (p.1090) (§15.4)

### IV

If

1. All limits of integration are constant
2. Integrand is of the form: \( f(x) \cdot g(y) \)

Then

\[ \int_{x=a}^{x=b} \int_{y=c}^{y=d} f(x)g(y) \, dy \, dx = \left( \int_{x=a}^{x=b} f(x) \, dx \right) \left( \int_{y=c}^{y=d} g(y) \, dy \right) \]

After class.

### Example

\[ \int_{x=0}^{x=2} \int_{y=1}^{y=3} \int_{z=2}^{z=4} xy^2 e^z \, dz \, dy \, dx \]

\[ = \left( \int_{x=0}^{x=2} x \, dx \right) \left( \int_{y=1}^{y=3} y^2 \, dy \right) \left( \int_{z=2}^{z=4} e^z \, dz \right) \]

\[ = \left( 2 \right) \left( \frac{8^3 - 1}{3} \right) \left( e^4 - e^2 \right) = \frac{1022}{3}(e^4 - e^2) \]

And if you get to the point where you do enough of these, often enough, you can "skip" the two steps and simply write down the answer.