II. Start §16.1: Line Integrals (p. 1127).

\[ I = \int_{x=a}^{x=b} f(x) \, dx \]

This indicates a line—namely the x-axis.

If \( f(x) \geq 0 \ \forall x \in [a, b] \), then we can think of \( I \) as the area "under the curve" from \( a \) to \( b \).

So if I wish to generalize my "line of integration" from a portion of the x-axis to, let us say, a portion of the curve \( y = x^2 \):

\[ \int_{x=a}^{x=b} f(x) \, dx \]

\[ \int_C f(x, y, z) \, ds \]

\[ z = f(x, y) \]
\[ z \geq 0 \]
**C. Evaluation Formula. (p. 1128)**

If \( \mathbf{w} = f(x,y,z) \) is continuous over a curve \( C \)

1. Find a **smooth** parametrization of \( C \)
   \( \mathbf{P}(t) = \langle g(t), h(t), k(t) \rangle \quad t \in [a, b] \).

2. **Eval. formula:**
   \[
   \int_C f(x,y,z) \, ds = \int_{t=a}^{t=b} f(g(t), h(t), k(t)) \|\mathbf{v}(t)\| \, dt
   \]

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**D. Two Examples:**

1. **Example 1:** p. 1132 #9. Eval. \( \int_C (x+y) \, ds \) \( W \)
   
   \( C: \ x = t \), \( y = 1-t \), \( z = 0 \). From \( (0,1,0) \) to \( (1,0,0) \).

   **Solution:**
   
   \[ \mathbf{P}(t) = \langle t, 1-t, 0 \rangle \]
   
   \( t = 0 \), \( 1 \) \( (\text{make sure this path STARTS at (0,1,0) and ENDS at (1,0,0)}) \)
   
   \[ \mathbf{v}(t) = \langle 1, -1, 0 \rangle \]
   
   \( \|\mathbf{v}(t)\| = \sqrt{2} \)

   \[ \int_C (x+y) \, ds = \int_{t=0}^{t=1} (t + (1-t)) \sqrt{2} \, dt \]
   
   \[ = \sqrt{2} \int_{t=0}^{t=1} dt = \sqrt{2} \]

2. **Example 2:** p. 1132 #11. Eval. \( \int_C (xy+y+z) \, ds \) along \( C: \mathbf{P}(t) = \langle 2t, t, 2-2t \rangle \), \( t = 0 \, \text{to} \, 1 \)

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\textbf{Solution:}\hspace{1cm} \textbf{1.} \overrightarrow{C} \text{ is already parametrized.} \\
\textbf{2.} \hspace{1cm} \overrightarrow{v}(t) = \langle 2, 1, -2 \rangle, \quad \|\overrightarrow{v}(t)\| = 3 \\
\textbf{3.} \hspace{1cm} \int_C (xy + y + z) \, ds = \int_{t=0}^{t=1} (2t^2 + t + (2-2t)) \cdot 3 \, dt \\
= 3 \int_{t=0}^{t=1} (2t^2 - t + 2) \, dt = 3 \left[ \frac{2}{3} t^3 - \frac{1}{2} t^2 + 2t \right]_{t=0}^{t=1} \\
= 3 \left( \frac{2}{3} - \frac{1}{2} + 2 \right) = 2 - \frac{3}{2} + 6 = \frac{16}{2} - \frac{3}{2} = \frac{13}{2} \\

\textbf{Note:} \hspace{1cm} \text{You won't always have } t = 0 \text{, and you won't always have the "speed" constant.}