POD #07 / §13.5 : p. 867 : #54 ; Find parametric equations for the line of intersection of the planes

\[ T_1: \ 2x + 5z + 3 = 0 \ \wedge \ T_2: \ x - 3y + z + 2 = 0 \]

Solution

1. The line \( L \) of intersection of the planes \( T_1 \) and \( T_2 \) will be orthogonal to the normal vectors of both planes.

   By the way, \( T_1 \) and \( T_2 \) are not parallel and they are not the same plane, as we shall verify in a moment, since \( \vec{n}_1 \times \vec{n}_2 \neq \vec{0} \), as we shall see. Thus \( L \) will be parallel to \( \vec{v} = \vec{n}_1 \times \vec{n}_2 \).

2. Computation:
   \[ \vec{v} = \vec{n}_1 \times \vec{n}_2 = \langle 2, 0, 5 \rangle \times \langle 1, -3, 1 \rangle = \langle 15, 3, -6 \rangle \]
   \[ = \langle 15, 3, -6 \rangle \]

3. Now we need one point \( P_0 \) on \( L \). So \( P_0 \in T_1 \) and \( P_0 \notin T_2 \).

   Let \( z = 0 \) : \( T_1: \ 2x + 3z = 0 \ \therefore \ x = -\frac{3}{2} \). Therefore,

   looking at \( T_2 \): \( (\frac{-3}{2}) - 3y + 0 + 2 = 0 \)
   \[ -3 - 3y + 0 = 0 \quad \therefore \ 1 = 6y \quad \therefore \ y = \frac{1}{6} \]

   \[ P_0 \left( -\frac{3}{2}, \frac{1}{6}, 0 \right) \in T_1 \cap T_2 \quad (\text{Mental check}) \]

4. Parametric Equations for \( L \in T_1 \cap T_2 \):

   \[ x = -\frac{3}{2} + 1t \\
   y = \frac{1}{6} + 3t \\
   z = 0 - 6t \]