13.1 Exercises

1. Suppose you start at the origin, move along the x-axis a distance of 4 units in the positive direction, and then move downward a distance of 3 units. What are the coordinates of your position?

2. Sketch the points (0, 5, 2), (4, 0, -1), (2, 4, 0), and (-1, -4, 1) on a single coordinate axes.

3. Which of the points P1(2, 3, 0), Q(1, -4, 0), and R(0, 2, 3) is closest to the xy-plane? Which point lies in the yz-plane?

4. What are the projections of the point (2, 3, 5) on the xy-, yz-, and xz-planes? Draw a rectangular box with the origin and (2, 3, 5) as opposite vertices and with its faces parallel to the coordinate planes. Label all vertices of the box. Find the length of the diagonal of the box.

5. Describe and sketch the surface in $R^3$ represented by the equation $x = -y$.

6. (a) What does the equation $x = 4$ represent in $R^3$? What does it represent in $R^2$? Illustrate with sketches.

(b) What does the equation $y = 3$ represent in $R^3$? What does it represent in $R^2$? Illustrate with sketches.

(c) What does the equation $z = 3$ represent in $R^3$? What does it represent in $R^2$? Illustrate with sketches.

(d) Find the length of the medians of the triangle with vertices $(1, 2, 3)$, $(4, 2, 0)$, and $(4, 1, 5)$.

7. Find the equation of a sphere if one of its diameters has endpoints (2, 1, 4) and (6, 3, 10).

8. Find equations of the spheres with center $(2, -3, 0)$ that touch
(a) the xy-plane,
(b) the yz-plane,
(c) the xz-plane.

9. Find an equation of the largest sphere with center $(3, 4, 9)$ that is contained in the first octant.

10. Show that the triangle with vertices $(1, 2, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$ is an equilateral triangle.

11. Find the lengths of the sides of the triangle with vertices $A(1, 2, 3)$, $B(4, 2, 0)$, and $C(4, 1, 5)$.

12. Show that the triangle with vertices $A(1, 2, 3)$, $B(4, 2, 0)$, and $C(4, 1, 5)$ is an equilateral triangle.

13. Describe in words the region of $R^3$ represented by the equation $x + y + z = 3$.

14. Find an equation of the sphere with center $(1, -4, 3)$ and radius 5. What is the intersection of this sphere with the xy-plane?

15. Find an equation of the sphere with center $(6, 5, -2)$ and radius $\sqrt{29}$. Describe its intersection with each of the coordinate planes.

16. Find an equation of the sphere that passes through the point $(1, 2, 3)$ and has center $(0, 1, 1)$.

17. Find an equation of the sphere that passes through the origin and whose center is $(2, 1, 3)$.

18. Show that the equation represents a sphere, and find its center and radius.

19. Prove that the midpoint of the line segment from $P(x_1, y_1, z_1)$ to $P(x_2, y_2, z_2)$ is

$$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2})$$

20. Find the lengths of the sides of the triangle with vertices $(1, 2, 3)$, $(4, 2, 0)$, and $(4, 1, 5)$.

21. Find an equation of the sphere with center $(2, 3, 5)$ that touches the xy-plane, the yz-plane, and the xz-plane.

22. Find an equation of the largest sphere with center $(3, 4, 9)$ that is contained in the first octant.

23. Find an equation of the sphere that passes through the point $(1, 2, 3)$ and has center $(0, 1, 1)$.

24. Find an equation of the sphere that passes through the origin and whose center is $(2, 1, 3)$.

25. Show that the equation represents a sphere, and find its center and radius.

26. Prove that the midpoint of the line segment from $P(x_1, y_1, z_1)$ to $P(x_2, y_2, z_2)$ is

$$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2})$$

27. Find an equation of the sphere with center $(1, -4, 3)$ and radius 5. What is the intersection of this sphere with the xy-plane?

28. Find an equation of the sphere with center $(6, 5, -2)$ and radius $\sqrt{29}$. Describe its intersection with each of the coordinate planes.

29. Find an equation of the sphere that passes through the point $(1, 2, 3)$ and has center $(0, 1, 1)$.

30. Find an equation of the sphere that passes through the origin and whose center is $(2, 1, 3)$.