II. Returned Test #1, No PID for Tue.

III. Begin 8.1.5 Functions of Several Variables, p. 923.

A. Example: Function to describe the volume of a rectangular box.

\[ V = \ell \cdot w \cdot h \]

This is a function of 3 independent variables \((\ell, w, h)\) and 1 dependent variable, \(V\).

B. Abstract Example: 

\[ f(x, y) = x^2 + y^2 \]

\[ z = x^2 + y^2 \]

III. How to "think" of functions.

A. A (mathematical) relation is a set of ordered pairs of numbers.

\[ \{ (1, 2), (3, -7) \} \quad \text{finite set.} \]

(A relation can be graphed)

\[ \{ x^2 + y^2 \} \quad \text{Graph:} \]

B. A function is a relation such that if \((x_1, y_1)\) and \((x_2, y_2)\)

are in \(\mathbb{R}\), \((x_1, y_1) \in R \text{ and } (x_2, y_2) \in R\), then

\[ y_1 = y_2 \quad \text{(Vertical line test.)} \]

D. Functions of 2 independent variables.

A function is a rule of correspondence. In this case it assigns to every

\((x, y) \in \text{Dom}\) exactly one \(z \in R_{y}\).

(See "Graph" p. 923)
IV. Let's look at some surfaces (Graphs of functions of 2 independent variables). See p. 928.

(c) Fig 10 \( f(x,y) = \sin(x) + \sin(y) \). 

We looked at problems in the book relating 3D surfaces w/ contour maps (level curve graphs). We also looked at MAPLE examples.

\[ \frac{2}{3} \]

\[ \text{MTH} \]

\[ \text{6:45 PM} \]

\[ \text{MARCH 15, 2013} \]

§ 15.2 Limits & Continuity, p. 938.

A. Refresh -- \( \varepsilon, \delta \) in Calc I.

Given \( y = f(x) \), \( x \in \mathbb{R} \). Let \( a \) be a point on the \( x \)-axis.

\[ \lim_{x \to a} f(x) = L \text{ means } \]

(i) Informally: As \( x \) gets closer-and-closer to \( a \), \( f(x) \) gets closer-and-closer to \( L \).

(ii) Verbally: to \( a \), \( f(x) \) gets closer-and-closer to \( L \).

(iii) Formal... (tomorrow).

\[ \text{to be continued...} \]