§2.5: p. 60: #8: Find the Gen. Sol. & find an interval on which it is defined.

\[ y' + 2xy = x^3 \]  \hspace{1cm} (*)

SOL: We previously started this problem in class, and we had found an integrating factor to be

\[ \mu(x) = e^{x^2} \]

(2) Next we “apply” the IF to b.s. of (*) and “collapse” the resulting LHS:

\[ e^{x^2}(y' + 2xy) = x^3e^{x^2} \]

\[ \therefore e^{x^2}y' + 2xe^{x^2}y = x^3e^{x^2} \]

\[ \therefore (e^{x^2}y)' = x^3e^{x^2} \]

\[ \therefore \int (e^{x^2}y)' \, dx = \int x^3e^{x^2} \, dx \]

(3) Now we integrate:

\[ e^{x^2}y = \int x^3e^{x^2} \, dx \]  \hspace{1cm} (##)

and we work out the RHS by (i) a subs, followed by (ii) int by pts.

(4) Given \( \int x^3e^{x^2} \, dx \)

\[ \therefore u = x^2 \Rightarrow du = 2x \, dx \]

or \( x \, dx = \frac{1}{2} \, du \)

\[ \therefore \int x^3e^{x^2} \, dx = \int u \, e^u \cdot \frac{1}{2} \, du \]

\[ = \frac{1}{2} \int \, u \, e^u \, du \]

\[ = \frac{1}{2} \left( ue^u - e^u \right) + C \]

\[ = \frac{1}{2} \left( x^2e^{x^2} - e^{x^2} \right) + C \]  \hspace{1cm} (####)

\[ \text{Foley's Method} \]

\[ \begin{array}{l}
\text{der} \quad \text{int} \\
\frac{e^u}{u} \quad \frac{e^u}{u} \\
\frac{e^u}{0} \quad \frac{e^u}{1} \\
\end{array} \]

\[ \text{cont...} \]