Finding the general solution and state an interval on which the general solution is defined:

\[ y' + \frac{1}{2} y = \cos(x) \]  

Use an integrating factor in the process of your solution.

**Solution:**

1. Find IF. There are two ways of doing this:
   - (a) Finding, analytically \( \mu(x) = e^{\int P \, dx} \)
   - (b) Asking the ODE—"What is your IF?" And then listening closely for an answer. At this stage of your learning process, I must always ask you to do method (a), but I hope that you will soon be able to use method (b). Here's how method (b) works:
     \[ x y' + y = (xy)' \]
     \( \mu(x) \) has just told me that \( \mu(x) = x \) is an IF.

But to get "credit" I must show that I can do method (a):

\[ \int P \, dx = \int \frac{dx}{x} = \ln|x| + C \]  

\[ \mu(x) = e^{\int P \, dx} = e^{\ln|x|} = x \]  

(\( \mu(x) = x \) for \( x > 0 \).)

2. Multiply both sides of (9) by \( \mu(x) \) to collapse the LHS and integrate:

\[ x (y' + \frac{1}{2} y) = x \cos(x) \]
\[ x y' + y = x \cos(x) \]
\[ (x y)' = x \cos(x) \]
\[ \int (x y)' = \int x \cos(x) \, dx \]  

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