II. Return PDE #05; Collect PDE #05; assign PDE #06; 82.4; p. 52; #18

[Diagram: A potential function]

III. Continue § 2.4: EXACT EQUATIONS, p. 46.

A. Example: 2.4: p. 52; #4: Determine if the ODE is EXACT. If it is EXACT, then SOLVE it.

(*) \((\sin(y) - y(\sin(x)))dx + (\cos(x) + x\cos(y) - y)dy = 0\)

Solve: 1st Test for EXACTNESS.

\[M(x,y) = \sin y - y\sin x, \quad N(x,y) = \cos x + x\cos y - y\]

\[\frac{\partial M}{\partial y} = \cos y - \sin x, \quad \frac{\partial N}{\partial x} = -\sin x + \cos y\]

\[\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}. \quad \therefore \text{The eq. (*) is EXACT.}\]

1. \(f(x,y) = \int M(x,y)dx + \text{another term} = \int (\sin y - y\sin x)dx = x\sin y + y\cos x + g(y)\)

2. \(f(x,y) = \int N(x,y)dy + \text{another term} = \int (\cos x + x\cos y - y)dy = \frac{x^2}{2} + C + g(y)\)

\[\therefore f(x,y) = x\sin y + y\cos x + \frac{x^2}{2} + C\]

The answer is ALWAYS \(f(x,y) = 0\).

III. Begin the VERY IMP. SECTION on LINEAR EQUATIONS, § 2.5: p. 54.

A. 1st Order Linear Ordinary Differential Equation (LODE)
What does it look like (in general)

\[ a_1(x) \frac{dy}{dx} + a_0(x) y = g(x) \quad (\text{E}) \]

If \( g(x) = 0 \), then

\[ a_1(x) \frac{dy}{dx} + a_0(x) y = 0 \quad (\text{F}) \]

is called a HOMOGENEOUS ODE.

We learn to solve (\text{F}) first.

IV. ODE POD 06: § 2.4: p 52; # 18.