I. I collected POD #10. There is no POD due tomorrow. Test #1 is next Mon, 9/04.

II. Begin 84.1.1 IVP $\mathfrak{v}$ BVP, p. 112.
For linear ODE. (LODE)

A. IVP Concrete Example.

\[
\begin{aligned}
x y'' + x^2 y' - 2e^x y &= \sin(x) \\
\text{ICs} &\quad y(1) = -2, \quad y'(1) = 5
\end{aligned}
\]

Initial value \[\{ \begin{array}{l} x_0 = 1, \\ y_0 = -2, \\ y'_0 = 5 \end{array} \]

B. Thm 4.1: (p. 112).

4 Background: \[ a_n(x) y^{(n)} + a_{n-1}(x) y^{(n-1)} + \ldots + a_0(x) y + a_{-1}(x) y = g(x) \]

\[ y(x_0) = y_0, \quad y'(x_0) = y'_0, \quad \ldots, \quad y^{(n-1)}(x_0) = y^{(n-1)}_0 \]

C. BVP. (p. 114) Example 2nd Order.

\[ a_2(x) y'' + a_1(x) y' + a_0(x) y = q(x) \]

Subject to: \[ y(a) = y_0 \; \mathfrak{v} \quad y(b) = y_1 \]

III. § 4.1.2: Linear Dependence/Independence (p. 115).

A. Defn: Linear Combination of Functions.
If \[ f_1(x), f_2(x), \ldots, f_n(x) \] are functions and if \[ c_1, c_2, \ldots, c_n \] are numbers (constants), then

\[ c_1 f_1(x) + c_2 f_2(x) + \ldots + c_n f_n(x) \]

is a linear combination of the functions.

B. Example: If \[ f_1(x) = x, \quad f_2(x) = x^2, \quad f_3(x) = 2x + 1 \]
then

\[ 2x - 5x^2 + 17(2x - 1) \]

is a linear combo. of the functions.

C. Question Arises: Do there exist numbers \[ c_1, c_2, c_3 \]
such that \[ c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x_0) = 0 \]?
If so, (follow-up question), what are those numbers?
D. If such numbers exist then the functions are called LINEARLY DEPENDENT. (or the set of functions \{f_1, f_2, f_3\} is a linearly dependent set).

E. In our case \( c_1 x + c_2 x^2 + c_3 (2x-1) = 0 \) (set \( L, f(x) = 0 \) solve for the \( c \)'s).

\[ \begin{align*}
\text{SOL} \quad \text{Re-write Lin Combo.} \\
 & \quad c_2 x^2 + (c_1 + 2c_3)x + (-c_3) = 0 \quad \text{(A)}
\end{align*} \]

\[ \begin{align*}
\therefore & \quad c_2 = 0 \quad \implies \quad c_2 = 0 \\
& \quad c_1 + 2c_3 = 0 \quad \implies \quad c_1 = 0 \\
& \quad -c_3 = 0 \quad \implies \quad c_3 = 0
\end{align*} \]

So the only way that the lin combo can "add up" to zero (see (A)) is for all the \( c \)'s to be zero. When this is the only way you can get a lin combo to be zero, then we say that the functions are LINEARLY INDEPENDENT.