I. No POD due tomorrow.

II. Example 23, p. 125. (§4.1.3-still!)
Given: \( y'' - 6y' + 11y - 6y = 0 \) (§)
\( \phi_1 = e^x, \phi_2 = e^{3x}, \phi_3 = e^{-2x} \) are solutions to (§)

Required: Find the Gen.Sol. to (§).

SOL: Using Thm 4.4, p. 122.

1. Consider
\[
W(\phi_1, \phi_2, \phi_3) = \det \begin{bmatrix}
e^x & e^{3x} & e^{-2x} \\
e^x & 3e^{2x} & 2e^{3x} \\
e^x & 4e^{2x} & 9e^{3x}
\end{bmatrix}
\]
\[
= (-1)^{4+2} e^x \begin{vmatrix}
2e^{2x} & 3e^{3x} \\
4e^{2x} & 9e^{3x}
\end{vmatrix} + (-1)^{4+3} e^{2x} \begin{vmatrix}
e^x & 3e^{3x} \\
e^x & 9e^{3x}
\end{vmatrix}
\]
\[
= e^x \left( 18e^{3x} - 12e^{5x} - 5e^{2x} \right) + e^{2x} \left( 3e^{3x} - 9e^{4x} \right)
\]
\[
= 6e^{6x} - 2e^{6x} + 2e^{6x} = 2e^{6x}
\]
\[ W(\phi_1, \phi_2, \phi_3) = 2e^{6x} \]
is not equal to zero \((-\infty, \infty) \) (R"): \( y'' - 6y' + 11y - 6y = 0 \)

3. \( y(x) = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} \) is the Gen.Sol. to (§)

III. p. 125 Non-Homogeneous Case.
Given a non-homogeneous LODE to solve, you break the process up into 2 MAJOR STEPS.

1. Consider the related (companion) homogeneous LODE, and find its Gen.Sol.
2. Find (by hook or by crook) any particular sol. to the non-homogeneous eq (original eq).

Then C Superimpose Them (i.e., Add them up).

IV. Example 24, p. 125a

Given: \( y'' + 9y = 27 \) (§)

Find a particular sol. to (§) \( y_p \)

Claim: \( y_p = 3 \)

(Proof of Claim: Verification) \( y_p = 0 \), \( y_p'' = 0 \).

2. \( y_p + 9y_p = 0 + 9(3) = 27 \)

Teacher's added note: \( \phi_1 = \sin(3x), \phi_2 = \cos(3x) \)

Teacher's claim: \( \phi_1 \) and \( \phi_2 \) are sols. to the complementary eq. \( y'' + 9y = 0 \)

1. \( \phi_1 = \sin(3x), \phi_2 = 3\cos(3x), \phi_2'' = -9\sin(3x) \).

2. \( y_p = 3\cos(3x) + 9\sin(3x) = 0 \)
Similarly, I can prove that $\phi_2 = \cos(3x)$ is also a sol. to the complementary LODE.

Use Wronskian to show $\{\phi_1, \phi_2\}$ is a fund set.

on $(-\infty, \infty)$

$$W(\sin(3x), \cos(3x)) = \det \begin{bmatrix} \sin(3x) & \cos(3x) \\ 3 \cos(3x) & -3 \sin(3x) \end{bmatrix}$$

$$= -3 \sin^2(3x) - 3 \cos^2(3x) = -3 \cdot 1 = -3 \neq 0 \text{ on } \mathbb{R}^1.$$

$\therefore \{\sin(3x), \cos(3x)\}$ is a fund set.

$\therefore$ The Gen Sol. to the complementary LODE is

$$Y_c = c_1 \sin(3x) + c_2 \cos(3x)$$

The Gen Sol. to $(*)$ is

$$Y = Y_c + y_p$$

$$Y = c_1 \sin(3x) + c_2 \cos(3x) + 3$$

Thus, what I have just done is "walk you through" the process of solving a non-homogeneous LODE.