FINAL EXAM is TOMORROW.

REVIEW:

A. Are these functions linearly dependent or linearly independent:
   \[ f_1(x) = 4 - x, \quad f_2(x) = 3x + 7, \quad f_3(x) = 4x. \]

   Background: Polynomials \( P_1(x) = P_2(x) \) What does that mean? If they are both third degree,
   \[ a_3x^3 + a_2x^2 + a_1x + a_0 = b_3x^3 + b_2x^2 + b_1x + b_0 \]
   So what does this mean?
   
   It means \( a_3 = b_3 \) and \( a_2 = b_2 \) and \( a_1 = b_1 \) and \( a_0 = b_0 \).
   (Corresponding coefficients MUST be equal.)

B. Systems of Equations (Solving),

Solve:
   \[ c_1(4-x) + c_2(3x+7) + c_3(4x) = 0 \]
   \[ 4c_1 - c_1x + 3c_2x + 7c_2 + 4c_3x = 0 \]
   \[ (-c_1 + 3c_2 + 4c_3)x + (4c_1 + 7c_2) = 0x + 0 \]
   \[ -c_1 + 3c_2 + 4c_3 = 0 \]
   \[ 4c_1 + 7c_2 = 0 \]
2. Solve \((2x - y) \, dx + (4y - x) \, dy = 0\) (*)

**Solution:**

1. **Method.** Ask if exact?
   
   \[ M(x, y) = 2x - y \quad N(x, y) = 4y - x \]
   
   \[ \frac{\partial M}{\partial y} = -1 \quad \frac{\partial N}{\partial x} = -1 \]
   
   \(\therefore\) (*) is exact.

2. \[ f(x, y) = \int M \, dx = \int (2x - y) \, dx \]
   
   \[ = x^2 - xy + g(y) \]
   
   **"Construct of Integrals"**

3. Because of **Exactness**
   
   \[ \frac{\partial f}{\partial y} = N(x, y) \]
   
   \[ \therefore -z + g'(y) = 4y - x \]

   **Conclusions:** \(g'(y) = 4y\) \(\Rightarrow\) \(g(y) = 2y^2 + C\)

4. \[ f(x, y) = x^2 - xy + 2y^2 + C \]

5. **The Solution** to (*) is \(f(x, y) = 0\) \(\therefore\)
   
   \[ x^2 - xy + 2y^2 + C = 0 \]