§ 4.1 Linear ODE's of Higher Order —
§ 4.1. Preliminary Theory
§ 4.1.1 My Example: IVP
\[ a_2(x)y'' + a_1(x)y' + a_0(x)y = g(x) \quad \text{AND} \]
\[ y(x_0) = y_0 \quad \text{and} \quad y'(x_0) = y'_0 \]

A Example:
\[ \sin(x) \frac{d^2y}{dx^2} - \cos(x) \frac{dy}{dx} + y = 7e^{-x} \]
and \( y(0) = 1, \frac{dy}{dx}(0) = 2 \).

B Theorem: 1 If all the coefficient functions AND the forcing function \( g(x) \) are continuous on an interval \( I \), and \( a_n(x) \neq 0 \ \forall \ x \in I \) and \( a_n(x) \neq 0 \ \forall \ x \in I \)

\( \forall = " \text{for each}," \ " \text{for every}," \ or \ " \text{for all}." \)

\( \exists = " \text{this is}," \ " \text{there are}," \ or \ " \text{there exists}," \)
there exists at least one.

AND 3 If \( x_0 \) is any number in \( I \)

THEN There exists a unique solution \( y(x) \) to the IVP on the interval \( I \).

C Solve: \( 2y'' + y' - 7y = 0, \ y(0) = 0, \ y'(0) = 0 \)

Solution: \( y(x) \equiv 0 \) (The zero function satisfies the ODE & the IC's. \( \Rightarrow \) By The Thm, \( y(x) = 0 \) is the only solution.)
D. $y''-y'-6y=0 \quad y(0)=0, \quad y(5)=7$
This is a BVP. It may or may not have a solution. It may or may not have a unique solution.

E. A HINT of THINGS to COME.
Consider $y''-y'-6y=0 \quad (*)$

**Solution**
1. Hummmmm! Try $y=e^{mx}$
2. $y'=me^{mx}$ and $y''=m^2e^{mx}$
3. $m^2e^{mx}-me^{mx}-6e^{mx}=0$
   $e^{mx}(m^2-m-6)=0$
   $e^{mx}(m-3)(m+2)=0$
   $m=3, -2$
4. This tells me that $y=e^{3x}$ and $y=e^{-2x}$ are both solutions to the original ODE $(*$)

F. What happens **IF**

$$y''+10y'+25y=0$$

Well the aux eq. is $m^2+10m+25=0$
This factors into $(m+5)^2=0 \quad m=-5 \quad \{-5\}^2$
Repeated root. $y=e^{-5x}$ is a solution, but are there any other solutions?