Discuss links on web page (Yesterday's Notes) to material on SHM.

Reviewed completion of problem in Yesterday's Notes.

§ 5.1: Simple Harmonic Motion

A. Definition of Motion (p. 186). (Read it!)

B. Alternate form of EoM.

1. Define $A$, the amplitude.

$$A = \sqrt{c_1^2 + c_2^2}$$

where

$$x(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t). \quad (\star)$$

2. Eq. (\star) can be re-expressed as

$$x(t) = A \sin(\omega t + \phi) \quad \text{(**)}$$

\[\text{amplitude} \quad \text{phase angle (p. 188)}\]

3. Period: $T = \frac{2\pi}{\omega}$

Frequency $f = \frac{1}{T} = \frac{\omega}{2\pi}$

4. Expand (\text{**}) by the Laws of Trig:

$$x(t) = A \sin(\omega t + \phi) = A \left[ \sin(\omega t) \cos(\phi) + \cos(\omega t) \sin(\phi) \right]$$

$$= A \cos(\phi) \sin(\omega t) + A \sin(\phi) \cos(\omega t)$$

$$= A \sin(\phi) \cos(\omega t) + A \cos(\phi) \sin(\omega t)$$
\[ c_1 = A \sin(\phi) \quad \text{and} \quad c_2 = A \cos(\phi) \]
and recall \[ A = \sqrt{c_1^2 + c_2^2} \]

\[ \sin(\phi) = \frac{c_1}{A} \quad \text{and} \quad \cos(\phi) = \frac{c_2}{A} \]

\[ \text{Look @ §5.1 \& p.191 \#25. (For SIHM)} \]

If \( x(0) = x_0 \) \& \( \dot{x}(0) = V_0 \), then
\[ A = \sqrt{x_0^2 + \left(\frac{V_0}{\omega}\right)^2} \]

\[ \text{So - if } x'' + \omega^2 x = 0 \quad x(0) = x_0, \dot{x}(0) = V_0 \]
\[ \text{eg } x'' + 2\omega x = 0 \quad x(0) = 1, \dot{x}(0) = 2 \]

According to \#25, \[ A = \sqrt{1^2 + \left(\frac{2}{\omega}\right)^2} = \sqrt{2} \]

\[ A = \sqrt{2} \]

Thus you can look at the "problem" and tell the Amplitude of the solution right away - even before you solve the problem!

\[ \text{Sol to \#25: The general to } x'' + \omega^2 x = 0 \text{ is} \]

1. \[ x = c_1 \cos(\omega t) + c_2 \sin(\omega t) \]

2. \[ \text{IC's: } x_0 = x(0) = c_1 \cos(0) + c_2 \sin(0) = c_1 \]

\[ \therefore \quad c_1 = x_0 \]

\[ V_0 = \dot{x}(0) = -c_1 \omega \sin(0) + c_2 \omega \cos(0) \]

\[ \therefore \quad V_0 = c_2 \omega \]

\[ A = \sqrt{c_1^2 + c_2^2} = \sqrt{x_0^2 + \left(\frac{V_0}{\omega}\right)^2} \]

\[ A = \sqrt{x_0^2 + \left(\frac{V_0}{\omega}\right)^2} \]
§ 5.2: Damped Motion. — Assume Damping Forces are proportional to $x(t)$ in particular assume that damping forces are $-\beta \dot{x}(t)$.

$\beta$ is constant $\beta > 0$.

Force acting on mass at any time $t$:

$$m \ddot{x} = -kx - \beta \dot{x}$$

Restorative spring force

$\Rightarrow \quad m \ddot{x} + \beta \dot{x} + kx = 0$

$$x + \frac{\beta}{m} \dot{x} + \frac{k}{m} x = 0$$

Call $\frac{k}{m} = \omega^2$

Now we decide to call $\frac{\beta}{m} = 2\lambda$ or $\lambda = \text{"Article" or "Lambda"}$

$$\Rightarrow \quad \ddot{x} + 2\lambda \dot{x} + \omega^2 x = 0 \quad (\star)$$

AuxEq. $m^2 + 2\lambda m + \omega^2 = 0$

Q.F. $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad m = \frac{-2\lambda \pm \sqrt{4\lambda^2 - 4\omega^2}}{2}$

$m = -\lambda \pm \sqrt{\lambda^2 - \omega^2}$

This $m$ is, of course, not mass, but it is the $m$ of the AuxEq.

Thus, knowing $(\star \star)$ we can look at $(\star)$ and almost immediately "see" $\lambda$ AND what the roots to the AuxEq are "like" — i.e. real or complex.