Problems 8.7.3. (p305)

A. Basic Function we are dealing with.

\[ f(t) \]
\[ f(t) \]
\[ a \quad b \quad t \]

(c) \( f(t)U(t-a) \) \( \leftrightarrow \) problem #45

(e) \( f(t)[U(t-a)-U(t-b)] \) \( \leftrightarrow \) prob. #46

Here was the PROBLEM!

47. Background

1. Example \( y = x^2 \)
   
   Compare 2. \( y = (x-1)^2 \)

   3. \( f(x) = x^2 \)
   4. \( f(x-1) = (x-1)^2 \)

2. Looking at the Basic function \( f(t) \) above,
   Consider \( f(t-a) \)

This is part of the vertically shifted function \( f(t-a) \)

(f) \( f(t-a)[U(t-a)-U(t-b)] \)

This "zeros" out everything before \( a \) and after \( b \).
II. Second Translation Theorem (§7.3: p.301)

Thm. 7.6 $\mathcal{L}\{f(t-a)U(t-a)\} = e^{-as}F(s)$

where $F(s) = \mathcal{L}\{f(t)\}$.

B. Corollary. If $f(t)=1$, then $f(t-a)=1$
and $\mathcal{L}\{f(t)\} = \frac{1}{s}$

So by S.T.I., $\mathcal{L}\{1 \cdot U(t-a)\}$

$= \mathcal{L}\{U(t-a)\} = \frac{e^{-as}}{s}$

C. Inverse Form of S.T.I. (p.302 Bottom)

$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)U(t-a)$

III. How does this stuff Work? Examples

A. My Example:

$\mathcal{L}\{(t-z)^2U(t-z)\} = e^{-2z} \frac{2}{s^3}$

B. Ex 7: $\mathcal{L}\{(t-z)^3U(t-z)\} = e^{-2z} \frac{6}{s^4}$
Write the function for this graph

\[ f(t) = 2 - 3u(t-2) + u(t-3) \]

Here's how I think he got it.

This part turns the "2" function on to begin with and then turns it off @ \( t = 2 \).

This part starts with the "-1" function off. Then it turns it on at time \( t = 2 \) and back off at time \( t = 3 \).

Put these two together and you get the desired graph.

And remove parentheses and "collect like terms" and you've got Zill's equation.

\[ f(t) = 2 - 3u(t-2) + u(t-3). \]