Conic Sections — We discussed Hypatia of Alexandria and the Conic Sections.

§10.1: p.744: Parabolas.

A You all know \( y = x^2 \) \( (y-k) = a(x-h)^2 \)

B The PARABOLA from a little different angle.

Three CONCEPTS: VFD

\[ \begin{align*}
V & : \text{vertex, a point} \\
F & : \text{focus, a point} \\
D & : \text{directrix, a line}
\end{align*} \]

\[ \begin{align*}
V(0,0) & \quad F(0,p) \\
& \quad \text{or} \\
& \quad F(p,0) \\
\text{for most of our work.} & \quad y = -p \quad \text{(horiz)} \\
& \quad \text{or} \\
& \quad x = -p \quad \text{(Vert.)}
\end{align*} \]

C Constructing a Parabola. By Ch 10 def — A parabola is the set of all points which are equidistant from a point (the focus) and a line (the directrix).
\[ D \] \textbf{Distance Formula in } \mathbb{R}^2 \text{ (the XY-plane).}

1. \[ d = \pm \sqrt{d_1^2 + d_2^2} \]
   \[ d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \]
   \text{DISTANCE FORMULA}

\[ d_1 = d_2 \] \text{ (PQ)} \text{ See the graph on the previous page.}

But \[ d_1 = d(FP) = \sqrt{(x-0)^2 + (y-p)^2} \]
and \[ d_2 = d(PQ) = \sqrt{(x-x)^2 + (y-(-p))^2} \].

Now it's a fact that \( A = B \) if \( A^2 = B^2 \).

Thus, it follows that \( d_1^2 = d_2^2 \). Therefore,
\[ (x-0)^2 + (y-p)^2 = (x-x)^2 + (y-(-p))^2 \]
\[ \Rightarrow x^2 + y^2 - 2py + p^2 = x^2 + (y+p)^2 = y^2 + 2py + p^2 \]
\[ \Rightarrow x^2 - 2py = 2py \]
\[ \Rightarrow x^2 = 4py \]

This is the "New Standard Form" (NSF) for a parabola with \( \text{V}(0,0) \) and \( \text{F}(0,p) \).