§ 9.8: Partial Fraction Decomposition (p. 715)

A
\[
\frac{2}{x+3} + \frac{5}{x-1} = \frac{2(x-1) + 5(x+3)}{(x+3)(x-1)} = \frac{7x + 13}{x^2 + 2x - 3}
\]

We know how to add (or subtract) two rational functions.

B

Look at inverse process.

Start with \( \frac{7x + 13}{x^2 + 2x - 3} \) and "break it apart."

\[
\frac{7x + 13}{x^2 + 2x - 3} = \frac{7x + 13}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}
\]

\[
\text{Sell } 7x + 13 = \frac{7x + 13}{x^2 + 2x - 3}
\]

\[
\text{So, } 7x + 13 = \frac{7x + 13}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}
\]

\[
\therefore A(x-1) + B(x+3) = 7x + 13
\]

3

\[\text{Two ways: (comparing polys)}\]

\[i\] \[\begin{align*}
(A+B)x + (-A+3B) &= 7x + 13 \\
\text{polynomials} &= \text{polynomial.}
\end{align*}\]

Two polynomials are equal if and only if corresponding coefficients are equal.

\[a_1x^2 + b_1x + c_1 = a_2x^2 + b_2x + c_2 \iff \begin{cases} a_1 = a_2 \\ b_1 = b_2 \\ c_1 = c_2 \end{cases} \]

\[a_1x + b_1 = a_2x + b_2 \iff [a_1 = a_2] \land [b_1 = b_2] \]

\[
\therefore A+B = 7 \quad \text{and} \quad -A+3B = 13
\]

\[
\frac{4B = 20}{4B = 20} \Rightarrow B = 5 \Rightarrow (A = 2)
\]

\[
\frac{7x + 13}{x^2 + 2x - 3} = \frac{2}{x+3} + \frac{5}{x-1}
\]
Technique — Subs of specific values

\[ A(x-1) + B(x+3) = 7x + 13 \]

\( \Box \) Let \( x = 1 \)

\[ A(1-1) + B(1+3) = 7(1) + 13 \]

\[ 4B = 20 \quad \Rightarrow \quad B = 5 \]

\( \Box \) Let \( x = -3 \)

\[ A(-3-1) + B(-3+3) = 7(-3) + 13 = -21 + 13 \]

\[ -4A = -8 \quad \Rightarrow \quad A = 2 \]

\( \Box \) Then finish

\[
\frac{7x + 13}{x^2 + 2x - 3} = \frac{2}{x+3} + \frac{5}{x-1}
\]