Just Do It. (Syn-Div.) (From §3.2)

A \[ \frac{3x^4 - 5x^3 + 2x - 10}{x + 4} = 3x^3 - 17x^2 + 68x - 270 + \frac{1070}{x + 4} \]

\[ \text{Sol':} \]

\[ \begin{array}{cccccc}
  & -4 & 3 & -5 & 0 & 2 & -10 \\
\hline
  & & -12 & 68 & -272 & 1080 \\
\hline
  & 3 & -17 & 68 & -270 & 1070 \\
\hline
\end{array} \]

\[ 3x^3 - 17x^2 + 68x - 270 \quad R 1070 \]

B Alt form of ANS.

\[ 3x^4 - 5x^3 + 2x - 10 = (x+4)(3x^3 - 17x^2 + 68x - 270) + 1070 \]

C Abstractly

1. \[ \frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)} \quad \text{Equivalent to} \]

2. \[ P(x) = D(x) \cdot Q(x) + R(x) \]

II Deal With §3.1 — As Review.

A Equality:

1. What does it mean for 2 fractions to be equal?
   
   i. Example: \[ \frac{1}{2} = \frac{3}{6} \quad \text{Why?} \]

   ii. Theory: \[ \frac{a}{b} = \frac{c}{d} \quad (a, b, c, d \text{ are integers and } b \neq 0, \; d \neq 0) \]

   \[ \frac{a}{b} = \frac{c}{d} \iff ad = bc \]

   \[ \frac{1}{2} = \frac{3}{6} \quad \Rightarrow \quad b'c' = 1.6 = 2.3 \]
This allows me to "reduce" fractions

\[ \frac{1}{2} = \frac{3}{6} \]

Now divide by 3

\[ \frac{3}{2} = \frac{6}{1} \]

\[ \frac{3}{6} = \frac{3 \cdot 1}{2 \cdot 2} \]

\[ \frac{3}{6} = \frac{1}{2} \]

This is what we do.

This is WHY (legally) we can do it.

What does it mean for two polynomials to be equal?

\[ 3x^2 - 2x + 5 = 3x^2 + bx + 5 \]

What is \( b \)?

\[ b = -2 \]

\[ 3x^2 - 2x + 5 = ax^4 + bx^2 + cx + 5 \]

\[ a = 0 \]

\[ b = 3 \]

\[ c = -2 \]

Back to §3.3 Concepts:

Evaluate \( P(-4) \) where

\[ P(x) = 3x^4 - 5x^3 + 2x - 10 \]

\[ P(-4) = ? \text{ a mess} \]

But

\[ P(x) = (x+4)(3x^3 - 17x^2 + 68x - 270) + 1070 \]

So

\[ P(-4) = 1070 \]

\[ \text{much easier!} \]

Continue with this tomorrow!