II. What About Something Like?

\[
\frac{6x^3 - 5x^2 + 10}{2x - 1} = \frac{3x^2 - x - \frac{1}{2} + \frac{19/2}{2x - 1}}
\]

Solution:

\[
\frac{6x^3 - 5x^2 + 10}{2x - 1} = \frac{2}{2} \cdot \frac{3x^3 - \frac{5}{2}x^2 + 5}{x - \frac{1}{2}}
\]

2. \[
\begin{array}{cccc}
3 & -\frac{5}{2} & 0 & 5 \\
\frac{3}{2} & -\frac{1}{2} & -\frac{1}{4} & \frac{19}{4} \\
3 & -1 & -\frac{1}{2} & \frac{19}{4}
\end{array}
\]

3. \[3x^3 - \frac{5}{2}x^2 + 5 = (x - \frac{1}{2})(3x^2 - x - \frac{1}{2}) + \frac{19}{4}\]

Multiply, b.s., by 2

4. \[6x^3 - 5x^2 + 10 = (2x - 1)(3x^2 - x - \frac{1}{2}) + \frac{19}{2}\]

III. Example of the Use of the Factor Theorem:

Divide \[\frac{x^3 + 3x^2 - x - 3}{x + 3}\]

Solution:

\[
\begin{array}{cccc}
-3 & 1 & 3 & -1 & -3 \\
-3 & 0 & 3 & 0 \\
1 & 0 & -1 & 0
\end{array}
\]

The point here is that the Remainder is ZERO!
OR

3. \[ x^3 + 3x^2 - x - 3 = (x+3)(x^2-1) \]

To Continue... (Complete factorization)

4. \[ x^2 - 1 = (x+1)(x-1) \]

5. \[ x^3 + 3x^2 - x - 3 = (x+3)(x+1)(x-1) \]

6. Which then leads to — The zeros of

\[ P(x) = x^3 + 3x^2 - x - 3 \] are \(-3, -1, 1\).

7. OR the roots of \( x^3 + 3x^2 - x - 3 = 0 \) are \(-3, -1, 1\).

IV. Find the Poly. such that \( \text{Deg } P = 3 \) and zeros are \( 2, -3, 7 \).

Soln. \[ P(x) = (x-2)(x+3)(x-7) \]
\[ = (x^2 + x - 6)(x - 7) = x^3 + x^2 - 6x - 7x + 42 \]
\[ P(x) = x^3 - 6x^2 - 13x + 42 \]

Now §3.3: p272: MAIN THING — RZT (Rational Zeros Theorem)

A. The IDEA — By Example —

1. Solve \( 3x - 2 = 0 \)

\[ 3x = 2 \]
\[ x = \frac{2}{3} \]

\[ \{2/3\} \]
What's really going on here is

\[ ax - b = 0 \]

\[ x = \frac{b}{a} \]

\( \{ \text{Leading Coeff} \} \)

\( \text{Constant Term} \)

What IS LHS?

1st degree Poly

3. The Poly \( P(x) = ax - b \)

has a zero of \( \frac{b}{a} \)

4. If \( P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \)

then ALL RATIONAL ZEROS (Fractions)

will look like this \( \frac{\text{factors of } a_0}{\text{factors of } a_n} \) or \( \frac{\text{factors of } b}{\text{factors of } a} \)

Example: List all possible Rat. Zeros of

(§ 3.3: p. 279: #4) \( S(x) = 6x^4 - x^2 + 2x + 12 \)

So \[ 1, -\frac{1}{1}, -\frac{2}{1}, -\frac{3}{1}, -\frac{4}{1}, -\frac{6}{1}, -\frac{12}{1} \]

So there are 24 possible Rat. Zeros of \( P(x) \).
It remains to check them out to see which are actually zeros.